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STRUCTURAL MECHANICS OF SUBMARINES

PART I

PRACTICAL METHODS AND EXAMPLES OF CALCULATIONS FOR THE HULL STRENGTH OF SUBMARINES

(Streitelynaya Mekhanika Podvodnykh Lodok)

HYDROMECHANICS

AERODYNAMICS

STRUCTURAL MECHANICS

APPLIED MATHEMATICS

ACOUSTICS AND VIBRATION

by

Yu. A. Shimanskiy
Leningrad

Translated by the Science Translation Service

STRUCTURAL MECHANICS LABORATORY

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PREFACE

The present book represents a part of a general treatise "Structural Mechanics of Ships," written for the Leningrad Institute of Ship Construction. Accordingly, the contents of the book and the character of presentation of the material, are correlated to the program and the development of this general course. Only those problems are considered in the book which are directly connected with the strength analysis of hulls of submarines, assuming that the reader is familiar with the fundamentals of general structural mechanics, which are used in the book or which are referred to.

The book is subdivided into two parts: in the first, the general foundation and the practical methods of strength analysis of submarine hulls of various design are discussed; in the second, the theoretical investigation of those problems is presented whose solutions are used in the first part of the book and which were not contained in other sections of the general treatise, or were not sufficiently developed in those sections.

In writing the first part, which is essentially of applied character, the author made use of the material presented by him in the third volume of the "Handbook of Ship Construction." Several of the author's published theoretical papers devoted to questions of structural mechanics of submarines were included in the second part of the book.

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INTRODUCTION

The hull of a submarine, from the point of view of structural mechanics, represents a hollow, riveted or welded beam of variable cross section which, in contrast to the hull of surface vessels, must possess not only a sufficient general longitudinal strength to equilibrate the weight forces and the water pressure distributed along its length, but also a sufficient general transverse strength to respond to large compressional effects caused by hydrostatic water pressure at the limiting depth of submergence of the submarine.

The construction of the hull of submarines and the performance characteristics of its various connections are so different from those of hulls of surface vessels that its strength analysis requires the application of quite different procedures and methods as compared with those which are already firmly established in structural mechanics of surface vessels. The procedures and methods peculiar to the strength analysis of submarines, besides their purely theoretical foundation, require in addition an experimental verification in order to establish the degree of reliability of the theoretical formulas and in order to introduce appropriate practical coefficients, as well as the specification of standards for design loads and allowable stresses.

Another outstanding peculiarity of strength analysis of submarine hulls is the fact that errors committed during construction may cause a sudden wreckage of the submarine at great depths of submergence, which precludes the possibility of not only preventive but also post facto detection of defects in submarine construction. This circumstance makes the performance characteristics of submarine hulls essentially different from those of surface vessels and other engineering structures.

In the world history of submarines, repeated wreckages have occurred from unknown causes, which are most probably explained by assuming the destruction of the hull at great depth due either to insufficient strength or accidental submergence beyond the limiting depth used in design.

The element of suddenness, pointed out above, in the destruction of the hull of submarines at great depths of submergence gives a particularly serious character of responsibility in ensuring its strength and therefore it is required to submit the completed hull to a hydrostatic test not only for checking the watertightness, but also to control the absence of some production defects or omissions during analysis.

If the hydraulic testing of the hull is performed in drydock by internal water pressure, then the forces and moments induced have the opposite sign as compared with those which will be present in normal operating conditions of the submarine and which entered into the analysis. This circumstance may require a supplementary reinforcement of some hull elements, or the mounting of special reinforcements for the time of hull testing by

internal water pressure.

The special importance and the peculiar character of strength analysis of submarines require the necessity for the establishment of a special course, devoted to their structural mechanics and containing both practical methods and procedures used in strength analysis of various elements of submarine hulls, as well as their theoretical and experimental justification.

In the present course only those problems of strength analysis of submarine hulls are considered which are absent in structural mechanics of surface vessels.

The practical methods and procedures of structural analysis of submarines are presented in the first part of the present course, and the derivation of the formulas in the second part. This subdivision was used with the aim to avoid an overloading of the first, main part with mathematical derivations at a loss of readability of the book.

PART I

PRACTICAL METHODS AND PROCEDURES OF STRENGTH ANALYSIS OF SUBMARINE HULLS

CHAPTER 1

RULES AND STANDARDS FOR THE STRENGTH ANALYSIS OF SUBMARINE HULLS

1. GENERAL FUNDAMENTALS FOR THE ESTABLISHMENT OF STRENGTH ANALYSIS OF SUBMARINE HULLS

1. The strength analysis of submarine hulls, of various designs as in the case of surface vessels, is conditional because of the necessity of making various assumptions or approximations, as, for example, in the determination of the magnitude of the load and in the development of the formulas which serve to determine the stresses which arise in the hull and to establish the instant at which the hull loses stability.

This inevitable inexactness of the conditional calculations is safeguarded by the introduction of factors of safety in accordance with previous experience with similar designs and with the results of full-scale or model experiments.

The factors of safety are introduced either into the design loading or into the allowable stresses. In the first case, the loading is considered as that which causes failure stresses: i.e., stresses for which there exists an impermissible deformation or which cause failure of the whole structure because of loss of stability. In the second case, the loading is considered as that for which the maximum stresses in the cross section must not exceed the standards established for allowable stress.

The suitability of one or the other method of conditional calculation depends on the conditions of each particular case. This problem, and also the general fundamentals which serve to establish the magnitude of the safety factors and the magnitude of the failure stresses, are considered in detail in the general course in Structural Mechanics of Ships; therefore, there is no necessity to establish them here.[1]

2. The above-mentioned general fundamentals, as applied to the strength calculations of surface vessels, serving to establish the factors of safety and the values of the failure stresses as a function of the nature of the loading and of the nature of the stresses which are produced, are set forth and summarized in the official publication "Requirements for

[1] References for Chapters 1 and 2 are listed on page 2-30.

Carrying Out Strength Calculations for Surface Vessel Hull Designs." This publication should also serve as a general guide for strength calculations for those structures of submarines whose purpose and mode of action is not unique to submarines. The requirements and standards for conditional strength calculations of submarine hulls, where they differ from the hull design of surface vessels, must be established in conformity with the specific aim and mode of action of the submarine designs. Such parts of the submarine as the so-called pressure hull are intended to receive and withstand the hydrostatic pressure of water at the limiting depth of submergence.

3. The requirements and standards relative to the strength calculation of specific structures of submarines must be established as a result of examination and analysis of the three basic problems of the structural mechanics of ships:[2]

The problem of the external forces - the determination of the magnitude and character of the externally acting forces.

The problem of the internal forces - the determination of the greatest forces and stresses in the cross sections of the structure arising from the assumed design loading.

The problem of the allowable stresses - the establishment, for the structure, of standards for the failure stresses as a function of the character of the stresses, and the establishment of the proper safety factor, considering the degree of conditionality as arbitrarily of the whole calculation and previous practical experience in the application of strength calculations to submarine structures.

The establishment of the ultimate strength of various designs of the pressure hull is the most difficult problem, the great importance of which stems from the peculiarly unfavorable conditions and consequences which accompany any impairment of strength in the submerged condition. The difficulty in solving this problem consists in the scant reliability of the applicable theoretical formulas determining the ultimate strength of the design and the necessity for introducing into these formulas correction coefficients which can only be determined from the results of full-scale or model tests of the structure.

4. There are developed, below, the general fundamentals for the establishment of the magnitude of the design load, i.e., the design depth of submergence for the entire pressure hull of the submarine, proceeding from the given limiting depth of submergence.

These fundamentals are used again in the following chapters in determining the standards and methods of analysis of various hull connections.

2. CHARACTER AND MAGNITUDE OF THE EXTERNAL LOAD ACTING ON THE HULL

A peculiarity of the design of the pressure hull of a submarine is that the strength of these designs is determined in large measure by their

stability and that the stresses increase more rapidly than the external load which causes them. This circumstance, as is known, must be taken into consideration by the introduction into the calculation of an assumed factor of safety for the magnitude of the loading and not for the magnitude of the allowable stress. The magnitude of the design load obtained by this method (i.e. the magnitude of the design depth of submergence) must be considered as the load at which the structure is in an unsafe condition, that is, in a condition near its ultimate strength, where the stresses approach the standards established for the failure stresses.

1. The load acting on the hull due to hydrostatic pressure (the intensity of loading is referred to the unit of length of the hull) can be resolved into the following two components (Figure 1):

(1) The load P, uniformly distributed, of intensity equal to

$$p = h \text{ tons per meter} \tag{1}$$

where p is the intensity of loading in tons per running meter along the circumference of the hull (for a portion of the length of the hull equal to 1 meter) and h is the depth of submergence of the axis of the hull in meters.

(2) The load P_1, distributed triangularly, of intensity

$$p_1 = r\cos\alpha \text{ tons per meter} \tag{2}$$

where r is the radius of the pressure hull in meters and α is an angle determining the position of points on the surface of the pressure hull.

The total component of the load p_1 (in the vertical direction, Figure 2)

$$P_1 = \int_0^{2\pi} p_1\, r\cos\alpha\, d\alpha = \pi r^2 \text{ tons} \tag{3}$$

i.e., equal to the displacement of a portion of the pressure hull 1 meter in length.

The load P, uniformly distributed, is completely balanced on the hull and, in the case of a circular hull, causes no bending forces.

The load P_1, distributed triangularly, must be balanced in a vertical direction by the forces of the weight of the submarine; forces of weight, balancing the pressure P_1, are set up for each portion of the length of the hull, partly in the form of forces of the weights P_2, existing in this portion, and partly in the form of the difference between the shearing forces P_3 acting at the boundary sections of the portion (Figure 3).

The aforesaid forces, i.e., the water pressure P_1, the weight forces P_2, and the shearing forces $P_3 = P_1 - P_2$, cause bending in the transverse cross section of the hull.

The distribution of the force P_2, related to the weights which are found directly in the section, depends on the weights and their location and can be established without difficulty for each particular case.

The law for the distribution of the force $P_3 = P_1 - P_2$ may be taken according to the known rule for the distribution of tangential forces in the transverse cross section of a beam of ring-like cross section under the action of shearing forces (Figure 4)

$$p_3 = \frac{P_3 \times S}{2I} = \frac{P_3 \times \left|2\int_0^{\alpha} r d\phi \times t \times r \cos\phi\right.}{2\pi r^3 t} = \frac{P_3}{\pi r} \sin\alpha \text{ tons per meter} \qquad [4]$$

where P_3 is the shearing force acting on the cross section

S is the static moment of the portion of the cross section lying on one side of a horizontal plane through the neutral axis

I $= \pi r^3 t$, the moment of inertia of the entire section relative to the neutral axis, and

t is the thickness of the plating of the pressure hull.

The direction of the stresses p_3, lying along the tangents to the boundary of the cross section as shown in Figure 4, is determined by the direction of the force P_3; consequently at different sections of the hull it will be different.

2. From the above considerations it follows that transverse sections of the hull of a submarine must be investigated for the action of the following loads:

(1) a uniformly distributed compressive loading

$$p = h \text{ tons per meter}$$

(2) a triangularly distributed loading

$$p_1 = r \cos\alpha \text{ tons per meter}$$

(3) a loading P_2, due to a part of the weight of the submarine, existing in the portion of its length which is under consideration;

(4) a loading $P_3 = \pi r^2 - P_2$, distributed according to the law

$$p_3 = \frac{P_3}{\pi r} \sin\alpha \text{ tons per meter}$$

The first of the above loadings, in the case of a true circular section, produces only uniform compression in the sections of the hull, causing compressive stresses equal (per meter of length of the hull) to

$$\frac{p \times 2r}{2} = pr \text{ tons}$$

To investigate the effect of the three other loadings, we consider the following two limiting cases:

(1) The loading P_2, due to the hull weight for the section of the length under consideration is absent, i.e., $P_2 = 0$;

(2) The loading P_2 is equal to the displacement of the section, i.e., $P_2 = \pi r^2$, and is applied as a concentrated force at the lower part of the cross section; in this case $P_3 = \pi r^2 - P_2 = 0$, and consequently the loading $p_3 = 0$.

It may be shown that, for a circular hull (Part 2, Chapter III), in the first case the action of the remaining three loadings does not cause bending moments in the cross section and only increases the compressive forces by the quantity

$$S_1 = r^2 \text{ tons} \tag{5}$$

In the second case (2) these loadings cause a maximum bending moment, equal to (in the lower section)

$$m = 0.75\, r^3 \text{ tons per meter} \tag{6}$$

and a compressive force, equal to

$$S_2 = 0.25\, r^2 \text{ tons} \tag{7}$$

Hence, the effect of non-uniform hydrostatic pressure on the transverse strength of a submarine hull and the nature of the distribution of the loadings (of the forces of weight) can become manifest in the increase of the axial (compressive) forces in the cross sections of the circular hull by an amount of the order of r^2 tons, and in the appearance of bending moments in these cross sections of the order of r^3 ton-meters.

From a comparison of the compressive forces r^2 with the axial force arising from the action of the loading p, equal to hr, it is evident that their ratio is equal to $r^2 : hr = r : h$, i.e., equal to the ratio of the radius of the hull to the depth of submergence.

Since the order of magnitude of this ratio for contemporary submarines is extremely small, not greater than 0.05, it then becomes possible, while not losing the practical accuracy of calculations for determining the axial forces in the circular cross section, to proceed with the design depth of submergence h taken to the centers of these cross sections.

The effect of the bending moments from the loading p_1 and from the forces of weight on the transverse strength of the hull can be substantial; this must be considered in calculating the strength of frames. It is possible to neglect this effect only in the case of the existence of sufficiently rigid longitudinal connections in the form of keels, foundations,

the plating and longitudinal location of the side and deck tanks and other structures which bind the frames to each other and to the stiff transverse bulkheads.

3. The external load which acts upon the pressure hull is the hydrostatic water pressure, the intensity of which varies with the depth of submergence, reaching its maximum value at the so-called limiting depth. The limiting depth is understood to be that maximum depth (to the axis of the hull) to which the submarine can repeatedly submerge (for example, lying upon the bottom), without any vestige of permanent deflection of its hull. Repeated submergence to the limiting depth can take place only a limited number of times, about 300 for the entire service life of the submarine, and then with a sufficiently large interval of time between each submergence. In this connection, one must consider the load, corresponding to the given limiting depth, in the nature of its variation, as constant; but in the nature of its action, as accidental.

Considering that at the limiting depth, the submarine may have trim, then one must take the load which acts at various transverse sections as variable, determining it according to the expression

$$h = h_{np} + \psi x, \qquad [8]$$

where h_{np} is the stated limiting depth of submergence,

ψ is the stated possible angle of trim, and

x is the distance of the cross section from amidships.

3. DETERMINATION OF THE DESIGN LOADING FOR THE PRESSURE HULL

As was shown in Section 1, the factor of safety which is used in the design of the pressure hull must be introduced into the load, in view of the absence of strict proportionality between the value of the external load and the stresses which it causes.

In order to completely ensure the transverse strength of the submarine hull, it is necessary, for proceeding from the acting load determined by Expression [8] to the design load, to take into account all circumstances which affect the magnitude of the safety factor. Analyzing these circumstances and considering certain of them in the following assignment of standards for the failure stresses, we may arrive at the conclusion that, for the transition from the acting load to the design load, one should:

a. increase somewhat the value of the stated limiting depth of submergence h_{np}, providing thereby for the possibility of an arbitrary over-submergence of a damaging character: this damaging over-submergence is to be taken as a certain fraction of the length of the submarine (about 20%).

b. to carry out the strength calculation according to formulas which determine the moment of its collapse (after loss of stability), introduce into the calculation a certain coefficient **k**, which takes into

account the fact that even before the inception of collapse of plating and frame, permanent deflections may appear, due, principally, to the not perfectly circular shape of the structure. The magnitude of this coefficient, however, can only be obtained on the basis of results of accurate experiments on the collapse of structures or their models.

A theoretical investigation of this problem leads us to conclude that the deviations from true circular shape in frames and plating have an extremely important effect upon the maximum compressive stresses which arise in them (see Chapter V of Part 2). Therefore the value of the coefficient must be selected as a function of the expected degree of accuracy of the assembling of the hull, and during the construction of submarines serious attention must be given to the control of the accuracy of its fabrication. Deviations from true circular shape in frames greater than 0.25 percent of the radius, and sag of the plating between frames greater than 15 percent of the plating thickness, cannot be permitted (Section 24).

On the basis of the above, the design loading, i.e., the design depth of submergence, must be determined in accordance with the following expression

$$h_p = k\,[h_{np} + 0.2\,L + \psi x], \qquad [9]$$

where k is a numerical coefficient, greater than unity, which must be established experimentally

L is the length of the submarine, and

h_{np}, ψ, x are quantities defined in Expression [8].

Assume, for example, that it is known that for a load equal to 70 percent of the collapse load h_p, which corresponds to the total loss of stability of the plating, there already begins to be evident local permanent sag in parts of the plating. The above-established basic condition of adequate strength of the plating, in this case, will consequently be

$$0.7\,h_p = [h_{np} + 0.2\,L + \psi x].$$

Dividing both sides of this equation by 0.7, we obtain the value of the coefficient k in Expression [9] as equal to

$$k = \frac{1}{0.7} = 1.43 = 1.4.$$

In this case, Expression [9], for the design depth, becomes

$$h_p = 1.4\,[h_{np} + 0.2\,L + \psi x]. \qquad [10]$$

According to Expression [10], the factor of safety, i.e., the ratio of the design depth to the limiting depth, at the midship section ($x = 0$), is obtained as

$$k = \frac{h_p}{h_{np}} = 1.4 \left[1 + \frac{0.2L}{h_{np}}\right] \qquad [11]$$

As can be seen, the factor of safety increases somewhat with an increase in the submarine's length and decreases somewhat with an increase in its limiting depth, which is quite proper.

In Figure 5 are shown graphically the relationships, derived above, between the limiting and the design depths of submergence for submarines 50 and 100 meters long, for a limiting depth of 100 meters; and for a submarine 100 meters long at a limiting depth of 150 meters. By dashed lines is shown the position of the submarine for the <u>operating</u> depth, taken as 2/3 of the limiting depth. By horizontal lines placed below the limiting depth, and down to the <u>design</u> depth, at which the hull can be crushed by water pressure, are shown the zones of forbidden depths of submergence for the submarine.

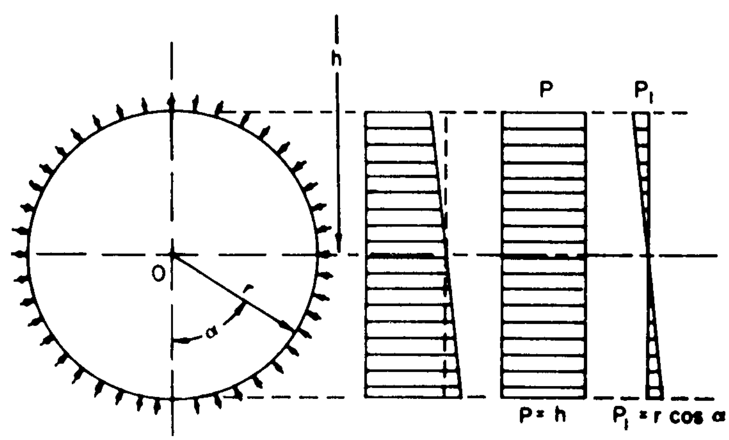

Figure 1

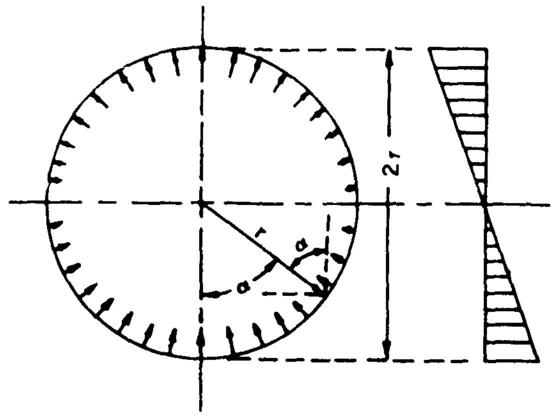

Figure 2

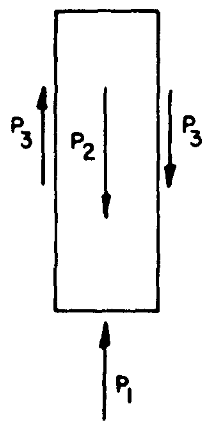

Figure 3

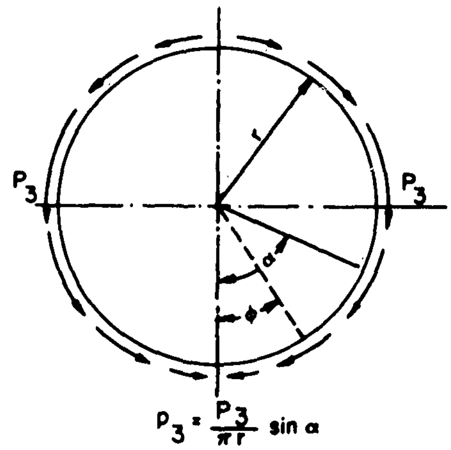

Figure 4

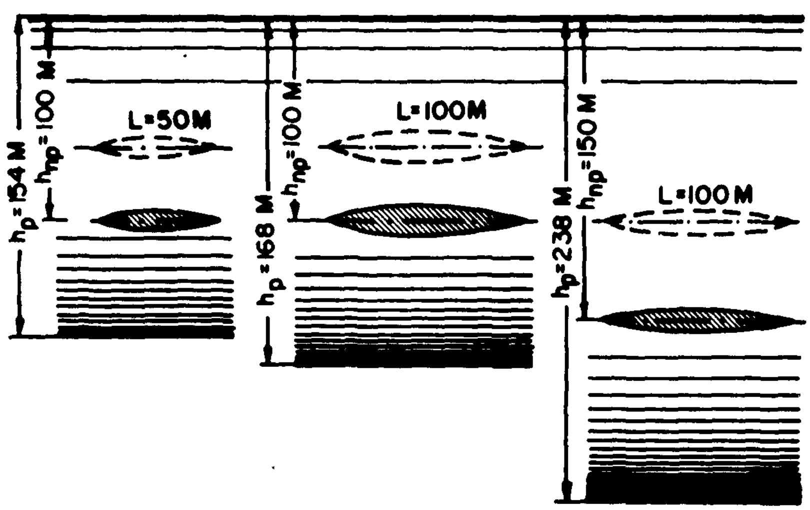

Figure 5

CHAPTER 2

STRENGTH ANALYSIS OF HULL PLATING

4. THE GENERAL CHARACTER OF THE DEFORMATION OF THE PLATING

1. We shall consider that the shell of the pressure hull of a submarine is cylindrical, and that the longitudinal curvature of the hull is negligible.

The cylindrical form of the shell can be either a true circle or any smooth curve approximating a circle. In the latter case, the radius of curvature of the shell, in the area under consideration can be taken as the radius of the circle which approximates the cylindrical surface in the area. In this way, in every case the calculation of the plating strength becomes a calculation of the strength of a thin cylindrical curved shell supported by transverse stiffeners (frames) and acted upon by water pressure distributed over its outer or inner surface. Besides the transverse load along the perimeter of the shell, longitudinal forces act as a consequence of the water pressure in the longitudinal direction.

The strength and nature of the deformation at collapse will vary depending upon whether or not the shell is acted upon by internal or external pressure. In the case of external pressure, collapse arises from loss of stability of the shell. This effect can occur at significantly lower water pressures than if the pressure acted on the inside. The essential difference between the external and internal pressures lies in the fact that for external pressure the strength of the shell is rendered more susceptible to the unavoidable (in practice) general and local deviations from a truly circular curve. A complete analogy exists, in this regard, with the deflection of a bar, having initial curvature, in tension and compression.

The resistance of the shell to internal pressure can be estimated with sufficient accuracy and certainty by using only theoretical formulas which give the value of the stress as a function of the dimensions and the pressure (Section 2, Part I of this book). For estimating the resistance of the shell to external pressure and for defining accurately the effect of the design dimensions, one must use, in addition, the results of full-scale and model experiments on shells, which must supplement and modify the corresponding theoretical formulas.

2. On the basis of the results of theoretical and experimental investigations of the resistance of shells to the action of external pressure of water, it is possible to draw the following picture of its deflection up to the moment of collapse.

Under the action of water pressure on all sides the shell undergoes first a deformation in compression which is symmetrical with respect to the axis; and a deformation in bending, in the span between frames. Under further pressure increase, the shell loses stability, with the formation on

it of observable humps and hollows; naturally at first these have a local character, and local humps and hollows can appear at a pressure considerably lower than the Euler (critical) pressure corresponding to the general loss of stability of the shell. The appearance of such premature local humps and hollows is explained by the presence in these places of minute structural deviations of the shell from true circular shape. The extent of these premature humps and hollows is limited by the frames; and therefore, for a sufficient reserve of local strength in the frames, even for a further increase in pressure, the humps and hollows preserve their local character. However, with the approach of the pressure towards the critical, the number of such local hollows and humps, and also their depth, increases, and they assume the character of undesirable permanent deformations in the shell.

The magnitude of the critical pressure is determined by the moment of rapid increase in the number of the local hollows and humps with their consequent transformation into a continuous wavy surface. Under conditions of insufficient local strength of the frames, a premature failure of the shell as a whole can occur as a result of the extension of any one of the local dimples beyond the limit of the interval between frames, with the formation of a dimple in that frame.

After the loss of stability of the shell, that is after general disappearance of its perfectly cylindrical shape, it is markedly deformed by even a small increase in pressure, accompanied by a sharp increase in the breadth and depth of the dimples between frames. From this moment, the shell in the vicinity of the dimple begins to act as an elastic plate, shirking the water pressure off onto the frames; therefore the possibility of further increasing the pressure is determined by the local strength of the frames. From a practical point of view, we may consider that the strength of the shell, already has begun to drop at this stage of the deformation; therefore, as the limiting strength of the shell, we should take that pressure corresponding to the inception of its general loss of stability.

During experimental investigations of the resistance of the shell to internal water pressure, this instant of its deformation is easily noted not only by visual observation of the condition of the hull, but also by the usual accompanying noise, and in addition by the sharp loss of water pressure on the manometer.

3. Full-scale and model tests have shown that the water pressure corresponding to the instant of stability loss, in actuality, is considerably smaller than that calculated by the theoretical formulas (Chapter 2 of Part II.)

This circumstance can be taken into account by the introduction of safety factors into the theoretical stability formulas, under conditions where the accuracy of construction of the shell is guaranteed by the establishment of production tolerances and where the stresses in the shell, at the critical pressure, are sufficiently removed from the elastic limit of

the material.

From the foregoing, it is evidently necessary to calculate the strength of the shell for stresses and for stability. Below is given a practical method for carrying out both of these calculations.

5. ANALYSIS OF STRESSES IN PLATING

1. The principal object of the stress analysis of the shell is to assure that the stresses arising in the cross sections of the shell at the critical pressure are not so great that the appropriate design formulas which determine the value of the critical load do not truly apply.

In this connection, practical standards must be established which cannot be exceeded.

The determination of the stresses in the shell at the critical load is conditional because at this load great stresses caused by the initial, essentially unavoidable out-of-roundness, arise which are not taken into account in the stress calculation. Having this in mind, it follows that the practical standards for stresses should be in conformity with the results of full-scale and model tests of shell stability. Analysis of the results of such tests leads us to the conclusion that we should standardize on the basis of the greatest stresses in the longitudinal cross sections of the middle surface of the shell. As a practical norm for these stresses, one should take a value not exceeding 80 percent of the elastic limit of the material. The greatest stresses in transverse sections of the shell at its supporting contour, which are local in character, can approach the elastic limit and even somewhat exceed it under the condition that the abovementioned stresses do not exceed this limit.

2. If the shell were not stiffened by frames and bulkheads, then the stresses in its transverse and longitudinal cross sections, at the mean surface, would be determined by the following expressions (Figure 6):

in the transverse sections

$$\sigma_1 = \frac{T}{t} = \frac{\pi r^2 q}{2\pi r t} = \frac{1}{2}\frac{qr}{t}; \qquad [1]$$

in the longitudinal sections

$$\sigma_2 = \frac{T_1}{t} = \frac{2rq}{2t} = \frac{qr}{t}, \qquad [2]$$

where q is the intensity of the external distributed pressure,

r is the radius of the shell, and

t is the thickness of the shell plating.

On account of the existence of transverse supporting members, apart from deflection due to compression, the shell will be subjected, in addition, to deflections of transverse and longitudinal bending in the in-

tervals between these members.* As regards the stresses in the longitudinal cross sections of the shell, in the vicinity of its stiffened sections they must evidently be smaller, as a consequence of the decrease of bending in that region. In the center of the span, however, these stresses can decrease or somewhat increase, in accordance with the spacing between transverse members. An increase of these stresses can arise on account of the unfavorable effect of the longitudinal forces upon the bending of the shell.

For the frame spacing generally accepted in practice, the maximum stress in the longitudinal cross sections of the shell becomes somewhat larger than that determined by Expression [2]; it does not, however, exceed the latter by more than 5 or 10 percent.

Although only the stresses in a longitudinal cross section have immediate practical significance in calculating the strength of the shell (which stresses can be obtained, with sufficient accuracy, by the elementary formula [2] after the introduction of a corrective coefficient of 1.05-1.10), nevertheless it is advisable to develop, as well, a more complete investigation of the deformation of the shell. In particular, the magnitude of the reactions acting between the shell and the frames, obtained in such an investigation, may have much practical interest. There is set forth below a complete investigation of the deformation of the shell which can be used, in addition, for estimating the effect of the degree of rigidity of the frames upon the stability of the shell.

3. The investigation of the deformation of a cylindrical shell, supported by uniformly spaced stiffeners (frames) and acted upon by a uniform external pressure, can be transformed into an investigation of an elementary strip of width equal to unity, cut from the shell by two meridional sections and acted upon by the following loads (Figure 7):

a. A load q uniformly distributed along its length.

b. A load, acting in the opposite direction, equal to T_1/r, representing the projection of the forces T_1, which are transmitted to the strip by the adjoining parts of the shell. These forces arise as a consequence of the compression of the shell ($E\frac{\omega}{r}t$) and as a consequence of the forces T in a perpendicular direction (μT).

The forces T_1 are equal to

$$T_1 = \mu \times T + E\frac{\omega}{r}t$$

and, consequently, the loading, acting on the elementary strip on the side opposite to the action of the loading q, will be

$$\frac{T_1}{r} = \frac{\mu T}{r} + \frac{E\omega t}{r^2};$$

*For the effect of the degree of stiffness of these members on the magnitude of the stresses in the shell, see Section 5 of Part II.

c. A compressive force in a longitudinal direction, equal to

$$T = \frac{\pi r^2 q}{2\pi r} = \frac{1}{2} qr$$

The effect of this force upon the bending of the strip, as is known, can be replaced by the action of a transverse distributed loading, of intensity $T\omega''$, where by ω is understood to be the equation of the elastic curve of the strip under consideration.

The differential equation of the bending of the strip is of the form

$$EI\omega^{IV} = q - \frac{Et}{r^2}\omega - \frac{\mu T}{r} - T\omega'' = q\left(1 - \frac{\mu}{2}\right) - \frac{Et}{r^2}\omega - \frac{qr}{2}\omega'', \qquad [3]$$

where $I = \dfrac{t^3}{12(1-\mu^2)}$, is the moment of inertia of the cross section of the strip.

The solution of this equation must satisfy the following boundary conditions (Figure 6):

a. The slope at the supports must be equal to zero, i.e., for $x = 0$ and $x = l$, $\omega' = 0$;

b. The displacement of the supports of the strip in the direction of the radius of the shell must be equal to

$$\omega_0 = \frac{Rr^2}{EF}$$

where R is the reaction of the supports of the strip, i.e., the intensity of the forces producing compression in the stiffener (frame); and

F is the cross-sectional area of the stiffener (frame).

The above equation is the differential equation of the bending of a beam lying on an elastic foundation which has a coefficient of stiffness Et/r^2, and is acted upon by a transverse loading $q(1-\mu/2)$ and a longitudinal loading $qr/2$.

The solution of this equation can be obtained either in closed form[3] or in the form of a trigonometric series.[4] Formulas for calculating the basic factors in the bending of the shell are given below. They are based upon a solution of Equation [3], obtained by trigonometric series; this solution is given in Section 2, Part II of this book.

4. Symbols (in units of kilograms and centimeters):

- r radius of the surface of the shell,
- t thickness of the shell,
- E Young's modulus ($E = 2x10^6$),
- μ Poisson's ratio ($\mu = 0.3$),
- l distance between frames,

F **cross-sectional area of the frame,**

q the loading ($q = 0.1h$, where h equals the depth of submergence, in meters).

$$\delta = \frac{t}{r}; \qquad \gamma = \frac{r}{l}; \qquad \left.\begin{array}{l} f = 0.85\,\dfrac{lt}{F}; \\[2ex] k = \dfrac{F}{F + lt} \\[2ex] \chi = \dfrac{k}{1 + 2kN} \end{array}\right\} \textbf{Assumed Notation}$$

The other quantities which enter **into the formulas, designated by** N, M, P, and Q, must be determined from Tables 2, 3, 4, and 5, **as a function of the values of the quantities a and c, which are calculated from the expressions**

$$a = 3.46\,\frac{\sqrt{rt}}{l}; \quad c = 0.062\,\sqrt[3]{\frac{q}{\gamma\delta}}; \sqrt{100\,\delta}$$

5. Formulas

The deflection of the shell at the frames (the deflection of the **frames)**

$$\omega_0 = \chi \frac{r}{E\delta} f q \qquad [4]$$

The **maximum** deflection of the shell

$$\omega_{max} = \chi \frac{r}{E\delta} (f + 3.4\,M) q \qquad [5]$$

The **reaction** of the shell on the **frames**

$$R = EF \frac{}{r^2}. \qquad [6]$$

The stresses at the **supporting** contour:

a. in the transverse cross sections of the shell

$$\sigma_1 = -\frac{q}{2\delta} \pm 37\,\chi\gamma^2 P q; \qquad [7]$$

b. in the longitudinal cross sections of the shell

$$\sigma_2 = -\frac{\omega_0}{r} E + 0.3\,\sigma_1 \qquad [8]$$

The stresses at mid-span:

a. in the transverse cross sections

$$\sigma_1{}' = -\frac{q}{2\delta} \pm 37\,\chi\gamma^2 Q q; \qquad [9]$$

b. in the longitudinal cross sections

$$\sigma_2' = -E\frac{\omega_{max}}{r} + 0.3\,\sigma_1' \qquad [10]$$

c. in the mean surface

$$\sigma_3' = -E\frac{\omega_{max}}{r} - 0.3\frac{q}{2\delta} \qquad [11]$$

In Expressions [7] and [9], the upper sign refers to the outer, and the lower, to the inner surface of the shell. For tensile stresses the sign is taken as plus; for compressive stresses, minus.

The design of the shell according to the above formulas is easily carried out using the tabular system of computation, as given in Table 1, applied to the following example.

Example. Find the stresses and the deflection in the shell of a submarine having the following characteristics:

Diameter of the pressure hull, 2r	400 centimeters
Thickness of the shell, t	13 millimeters
Frame spacing, l	700 millimeters
Design depth of submergence, h	150 meters
Frame, channel bar No. 16	(F = 24.9 centimeters^2)

We calculate and enter in Table 1 the following quantities

$r = 200$ cm; $t = 1.3$ cm; $l = 70$ cm; $F = 24.9\ \text{cm}^2$; $lt = 70 \times 1.3 = 91.0\ \text{cm}^2$;

$F + lt = 24.9 + 91.0 = 115.9\ \text{cm}^2$;

$$\delta = \frac{t}{r} = \frac{1.3}{200} = 6.5 \times 10^{-3};\ \sqrt{100\,\delta} = 0.805;$$

$$f = 0.85\frac{lt}{F} = 0.85\frac{91.0}{24.9} = 3.1;$$

$$\gamma = \frac{r}{l} = \frac{200}{70} = 2.86;$$

$$\gamma^2 = 2.86^2 = 8.15;$$

$$k = \frac{F}{F + lt} = \frac{24.9}{115.9} = 0.215\,;$$

$q = 0.1 \times 150 = 15$ atm*;

$$E = 2 \times 10^6\ \text{kg/cm}^2;\ \sqrt[3]{\frac{q}{\gamma\delta}} = \sqrt[3]{\frac{15}{6.5 \times 10^{-3} \times 2.86}} = 9.31;$$

$$a = 3.46\frac{\sqrt{tr}}{l} = 3.46\frac{\sqrt{1.3 \times 200}}{70} = 0.795;$$

$$c = 0.062\sqrt[3]{\frac{q}{\gamma\delta}} : \sqrt{100\,\delta} = \frac{0.062 \times 9.31}{0.805} = 0.715$$

*Editor's Note: the abbreviation for "atm" (atmospheres) is written, in Russian script, as am.

From the derived values of the quantities a and c we find, using Tables 2, 3, 4, and 5, the values of N, M, P, and Q:

$$N = 1.025;\ M = 0.863;\ P = 2.44;\ Q = -0.40$$

We develop the computation of the deflections and the stresses in the shell according to Formulas [4] through [11], using the corresponding lines in Table 1.

Line 1. $$2kN = 2 \times 0.215 \times 1.025 = 0.440$$

Line 2. $$\chi = \frac{k}{1 + 2kN} = \frac{0.215}{1 + 0.440} = 0.149$$

Line 3. $$37\,\chi\gamma^2 q = 37 \times 8.15 \times 15 \times 0.149 = 674$$

Line 4. $$37\,\chi\gamma^2 qP = 674 \times 2.44 = 1640$$ (The second member of formula 7)

Line 5. $$37\,\chi\gamma^2 qQ = -674 \times 0.40 = -270$$ (The second member of formula 9)

Line 6. $$\frac{q}{2\delta} = \frac{15}{2 \times 6.5 \times 10^{-3}} = 1155$$ (The first member of the formulae 7 and 9)

Line 7. $$3.4\,M = 3.4 \times 0.863 = 2.93$$

Line 8. $$f + 3.4\,M = 3.1 + 2.93 = 6.03$$

Line 9. $$\chi\frac{rq}{E\delta} = 0.149\,\frac{200 \times 15}{2 \times 10^6 \times 6.5 \times 10^{-3}} = 0.0344$$

Line 10. The deflection of the frame according to Formula [4]

$$\omega_0 = \chi\frac{rq}{E\delta} f = 0.0344 \times 3.10 = 0.107 \text{ cm}$$

Line 11. The maximum deflection of the shell by Formula [5]

$$\omega_{max} = \chi\frac{rq}{E\delta}(f + 3.4\,M) = 0.0344 \times 6.03 = 0.208 \text{ cm}$$

Line 12. The reaction of the shell on the frame by Formula [6]

$$R = EF\frac{\omega_0}{r^2} = \frac{2 \times 10^6 \times 24.9 \times 0.107}{200^2} = 134 \text{ kg/cm}$$

Line 13. The stress in the cross section of the frame

$$\sigma = -E\frac{\omega_0}{r} = -2 \times 10^6\,\frac{0.107}{200} = -1070 \text{ atm}$$

Line 14. The stress at the inner surface of the shell in a transverse cross section at the frame, by Formula [7]

$$\sigma_1 = -\frac{q}{2\delta} - 37\ \chi\gamma^2\ qP = -1155 - 1640 = -2795 \text{ atm}$$

Line 15. The same at the outer (loaded) surface of the shell, by Formula (7)

$$\sigma_1 = -1155 + 1640 = +485 \text{ atm}$$

Line 16. The stress at the inner surface of the shell in a longitudinal cross section, at the frame, by Formula [8]

$$\sigma_2 = -E\frac{\omega_0}{r} + 0.3\ \sigma_1 = -1070 - 0.3 \times 2795 = -1905 \text{ atm}$$

Line 17. The same at the outer surface of the shell, by Formula [8]

$$\sigma_2 = -1070 + 0.3 \times 485 = -925 \text{ atm}$$

Line 18. The stress at the inner surface in a transverse cross section at mid-span, by Formula [9]

$$\sigma_1' = -\frac{q}{2\delta} - 37\ \chi\gamma^2\ qQ = -1155 - (-270) = -885 \text{ atm}$$

Line 19. The same at the outer surface, by Formula [9]

$$\sigma_1' = -1155 + (-270) = -1425 \text{ atm}$$

Line 20. The stress at the inner surface of the shell in a longitudinal cross section at mid-span, by Formula [10]

$$\sigma_2' = -E\frac{\omega_{max}}{r} + 0.3\,\sigma_1' = -1070\frac{0.208}{0.107} - 0.3 \times 885 = -2080 - 265 = -2345 \text{ atm}$$

Line 21. The same at the outer surface, by Formula [10]

$$\sigma_2' = -2080 - 0.3 \times 1425 = -2505 \text{ atm}$$

Line 22. The stress at the mean surface of the shell in a longitudinal cross section at mid-span, by Formula [11]

$$\sigma_3' = -E\frac{\omega_{max}}{r} - 0.3\frac{q}{2\delta} = -1070\frac{0.208}{0.107} - 0.3 \times 1155 = -2425 \text{ atm}$$

6. STABILITY ANALYSIS OF PLATING

1. The calculation of the shell of the submarine's hull for stability

is essentially the problem of investigating the stability of a thin cylindrical shell, supported by stiffeners of finite rigidity (frames), acted upon by an external uniformly distributed pressure, and in addition by longitudinal forces arising from the action of that pressure upon the transverse partitions of the shell.

This problem is one of the most complex problems of structural mechanics, not only because of the mathematical difficulties associated with its accurate solution, but, mainly, because of the necessity for introducing extremely important practical correction factors, into the results obtained by theoretical means, for the purpose of bringing the solution into agreement with the results of direct experiment. The principal reasons which explain the large deviation between theory and practice are the following (see Section 33 of Part II):

a. Under conditions of the existence of the high stresses which arise in the shell at the moment of its loss of stability, the deformation curve of the material is considerably different from the curve which is used in the derivation of the theoretical formulas.

b. In view of the existence of essentially unavoidable initial deflection in the shell, which is a consequence of a deviation from true circular shape, under conditions of compression, there arise large additional stresses not taken into account by the theoretical formulas.[5]

For these reasons, no theoretical formula can be recommended, as long as it is not in agreement with the results of direct experience and is not adjusted to be so; on the other hand, any of the known theoretical formulas, under that condition, become almost equally reliable within the limits for which they have been adjusted.

2. One of the first papers dealing with the question of the stability of cylindrical shells supported by stiffeners, was that of Lorenz.[6] His formula was simplified by I. G. Bubnov, and, for the case of a large number of waves around the circumference of the shell, was presented in the following form.[7]

$$q_p = \frac{E \times u}{1-\mu^2}\left(\frac{t}{r}\right)^2 \times K \qquad [12]$$

with

$$n = k\sqrt{\frac{r \times u}{t}}$$

$$u = \frac{\sqrt{rt}}{l}$$

where q_p is the critical value of the uniformly distributed external pressure;
t is the thickness of the shell;
r is the radius of the shell;

n is the number of waves around the circumference, at the time of loss of stability;

u is an argument;

l is the spacing between frames;

K,k is the numerical coefficients, which are a function of u and are determined from Table 6.

Subsequently, the theoretical formula of Southwell appeared in 1913.[8]

$$q_{\text{э}} = E\frac{t}{r}\left[\frac{\pi^4}{n^4(n^3-1)}\frac{r^4}{l^4} + \frac{n^2-1}{12(1-\mu^2)}\frac{t^2}{r^2}\right] \quad [13]$$

In 1914 von Mises published his formula, which he derived for calculating the stability of fire tubes, i.e., without considering longitudinal compression.[9]

Much later von Mises published a second formula taking into account longitudinal compression.[10] This second formula may be put into the following form:

$$q_{\text{э}} = E\frac{t}{r}\left\{\frac{1}{n^2\left(1+\frac{n^2l^2}{\pi^2r^2}\right)^2} + \frac{n^2}{12(1-\mu^2)}\frac{t^2}{r^2}\left[1+\frac{\pi^2r^2}{n^2l^2}\right]^2\right\}\times\left(1+\frac{\pi^2r^2}{2n^2l^2}\right)^{-1} \quad [14]$$

In Formulas [13] and [14] the integer n signifies the number of waves which appear around the circumference of the shell at the instant of loss of stability; this number n must be taken as that for which the quantity $q_{\text{э}}$ in the formulas has a minimum value (n > 2)

The critical pressure $q_{\text{э}}$, as determined by von Mises' second formula (Equation 14), and also the corresponding number of waves n, can be found by the use of the curves and formulas given in Figure 8.

Since in practice, the number n is generally fairly large (of the order of 15), there exists the possibility of excluding it from formulas [13] and [14] by means of investigating an analytical minimum for $q_{\text{э}}$ as a function of the variable n. This improvement on these formulas, carried out by P. F. Papkovich,[11] transforms them into the following:

a. Southwell's formula

$$q_{\text{э}} = 18.8\left(\frac{100\,t}{r}\right)^{3/2}\times\left(\frac{100\,t}{l}\right) = 18.8\times10^5\,\frac{t^{2.5}}{r^{1.5}l} \quad [15]$$

b. von Mises' second formula

$$q_{\text{э}} = 19.1\left(\frac{100\,t}{r}\right)^2\times\left(\frac{100\,tr}{l^2}\right)^{0.58} \quad [16]$$

where r is the radius of the circumference of the shell

t is the thickness of the shell

l is the spacing between frames.

It should be noted that in developing all the theoretical formulas for stability, due to the mathematical complexity of obtaining an exact solution of the problem, various important assumptions are made. Thus, in particular, Southwell's solution was obtained on the assumption of the absence of a loading acting longitudinally on the shell; in his solution, as in all known solutions, the stiffeners are assumed to be acting only so as to prevent a change in the shape of the shell, and the shell itself is assumed to be completely inextensible.

There is given below the solution of this problem derived without the introduction of the foregoing assumptions. This solution may be obtained in a simple and more lucid manner, by applying the method of potential energy[12] (see Chapter 2 of Part II of this book).

Symbols:

r is the radius of the cylindrical shell

t is the thickness of the shell

l is the spacing between frames

$$E_1 = \frac{E}{1-\mu^2}$$

$$\left.\begin{aligned} \delta &= \frac{t}{r} \\ \gamma &= \frac{r}{l} \\ \alpha &= \pi\gamma \end{aligned}\right\} \text{Assumed Notation}$$

n is the number of waves along the circumference at the time of loss of stability.

χ is a coefficient which characterizes the effect of the degree of stiffness of the frames upon the stability (see the foregoing Section 5).

When subjected to a uniform all-sided external pressure q, the shell loses stability at a pressure determined by the expression:

$$q_{\flat} = \frac{2\delta \times E_1}{2(1-0.85\,\chi)n^2 + \alpha^2}\left[\frac{\alpha^2}{3n^2 + \alpha^2} + \frac{\delta^2}{12}\,(n^2 + \alpha^2)^2\right] \qquad [17]$$

The integer n in this expression must be taken as that for which $q_{\flat}$ has the least value.

For the usual assumption that the stiffeners do not prevent shrinkage [compression] of the shell, the coefficient χ must be taken as zero.

Using Table 7, the value of the integer n as a function of δ and γ may be found. This is to be substituted in Formula [17].

In Table 8 are given the values of the critical pressure $q_{\flat}$ according to Equation [17] as a function of the values of δ and γ (for $\chi = 0$). From the data of this Table, the corresponding curves shown in Figure 9 were constructed.

3. Bearing in mind the necessity, noted above, for introducing into the theoretical formulas for stability correction factors as indicated in Section 1, we must take the following practical method of calculating the stability of the submarine's shell:

a. Calculate, according to the three Formulas [15], [16], and [17], the critical pressure q_p.

b. Find from Table 9 the values of the correction factors η, as a function of the design thickness of the shell t in millimeters, which take account of the effect of initial deflection.

c. Calculate the average compressive stress in the longitudinal cross sections of the shell, corresponding to a pressure $\eta_1 q_p$, by the approximate formula

$$\sigma_p = 1.1 \times \eta_1 q_p \frac{r}{t} \; ; \qquad [18]$$

d. Find from Table 10 the value of the correction factor η_2, which takes into consideration the effect of the stresses upon the stability.

e. Calculate the actual critical pressure for the shell q_{1p} from the formula

$$q_{1p} = \eta_1 \eta_2 q_p \qquad [19]$$

f. Calculate the average compressive stress in the longitudinal cross section, corresponding to a pressure q_{1p} by the formula

$$\sigma_{1p} = \eta_2 \times \sigma_p \qquad [20]$$

This stress is to be compared with the established norm for permissible stress, which is equal to $0.8\sigma_T$.

The above stability analysis should be carried out in the tabular form shown in Table 11. In carrying out the computations, the following should be taken into consideration:

a. Since the actual shell thickness may differ from the specified thickness, and having in mind the great effect of thickness upon stability, it is recommended that a thickness t be used, which is less than the nominal thickness t_1; the value of t should be selected from considerations of the technical conditions under which the thickness of steel is measured. In general, take the thickness t as $0.96\ t_1$.

b. The critical pressure should be taken as the least of the values obtained from the three Formulas [15], [16] and [17].

Example. Find the critical pressure for the shell of a submarine having the following characteristics:

Diameter of the pressure hull, $2r$	4 meters
Nominal thickness of the shell, t_1	1.3 cm.
Spacing between frames, l	700 mm.
Cross sectional area of the frame (channel bar Number 16)	24.9 cm.^{2}

Material:

High tensile steel:

$E = 2 \times 10^6$ kg./cm.^{2} [metric atmospheres]

$\mu = 0.3$

$\sigma_T = 3{,}000$ kg./cm.^{2}

The computation is carried out using Table 11, in which are entered the results of the computations for this example. The value of the quantity χ is taken from the calculation of the stresses in the shell made in the previous example (Section 5).

In the first lines of the table the values of the following quantities are entered:

Nominal shell thickness, t_1	1.3 cm
Radius of the shell, r	200 cm
Frame spacing, l	70 cm
Design shell thickness, $t = 0.96\, t_1$	1.25 cm

Ratios:

$$\gamma = \frac{r}{l} = \frac{200}{70} = 2.86\ ;$$

$$\delta = \frac{t}{r} = 6.25 \times 10^{-3}$$

We develop the computation of stability by Formula [17] in the following order:

a. Determine the number of waves n, at which the shell collapses due to loss of stability. The number n is found from Table 7; for the given values of γ and δ. ($n = 16$).

b. Calculate the quantity $\alpha^2 = \pi^2\gamma^2$, which enters into Formula [17]. $\alpha^2 = 80.7$

c. Calculate the value of the coefficient χ, which takes into account the effect of the degree of stiffness of the frames upon the shell's stability ($\chi = 0.149$ from Table 1, line 2).

d. Determine the value of the correction factor η_1 from Table 9; for a thickness $t = 12.5$ mm., $\eta_1 = 0.79$.

e. Compute from Formula [17] the theoretical value of the critical pressure q_9; in Table 11, E is taken as 2×10^6 kg./cm.^{2} for Formula [17].

f. Determine the average stress σ_9 in the longitudinal cross section of the shell at mid-span, corresponding to a pressure q_9, according to the approximate Formula [18].

g. Find from Table 10 the value of the correction factor η_2,

which takes into account the effect of the magnitude of the stress upon the shell's stability (for $\sigma_{э} = 3340$ kg./cm.^{2}, $\eta_2 = 0.77$).

h. Compute the design value of the critical pressure from Equation [19].

$$q_{1э} = \eta_2 \eta_1 q_{э}$$

i. Compute the average stress in the longitudinal cross sections of the shell, which corresponds to the critical design pressure, according to Equation [20].

j. In the last part of Table 11, the computation, in a similar way, of the critical pressure by Formulas [15] and [16] is given.

7. APPROXIMATE DETERMINATION OF THE THICKNESS OF THE SHELL PLATING AND OF THE SPACING BETWEEN FRAMES

1. The optimum design of the submarine's hull, from the standpoint of weight, must approach that for which the average compressive stress in the shell (in a longitudinal cross section at mid-span) at the moment of loss of stability, is equal to the assumed standard for allowable stress in the material,

$$\sigma_\partial = 0.8 \times \sigma_T$$

This average compressive stress can be obtained from the following approximate formula, with sufficient accuracy for present purposes:

$$\sigma = 1.1 \frac{q_{1_\flat} r}{t} \qquad [21]$$

After substituting in the left side of this expression the assumed allowable stress, and solving for t, we have the following expression for the shell thickness, for which the stress will not exceed the standard:

$$t = \frac{1.1 \times q_{1_\flat} r}{0.8 \times \sigma_r} = 1.4 \frac{q_{1_\flat} r}{\sigma_T} \qquad [22]$$

2. Having computed from Equation [22] the shell thickness t, the frame spacing l corresponding to this thickness may easily be found. Then the critical pressure will be equal to the stated value of $q_{1_\flat}$, using Equation [19], which established the relation between the actual critical pressure and the critical pressure calculated from the theoretical Formula [$q_\flat$]

$$q_\flat = \frac{1}{\eta_1 \eta_2} \times q_{1_\flat} \qquad [23]$$

The magnitude of the correction factor η_1, must be taken from Table 9 for the already established thickness t. In case theoretical Formula [17] is used, the correction factor η_1, must in addition be obtained according to another expression (Formula [5], Section 34 of Part II).

$$\eta_1 = \left(1 - \frac{0.3}{t}\right) \qquad [24]$$

where t is the shell thickness in centimeters.

The value of correction factor η_2 must be taken from Table 10 for a stress (corrected), equal to $0.8\sigma_T$.

Finding the theoretical value of the critical pressure $q_\flat$ from Equation [23] and knowing the value of $\beta = t/r$, we can, using Table 8 or the

diagram in Figure 9, find the corresponding value of $\gamma = r/l$, from which the frame spacing becomes

$$l = \frac{r}{\gamma} \qquad [25]$$

Example 1. Find the shell thickness t and the frame spacing l for a submarine with shell radius $r = 200$ cm. with a given critical pressure $q_{1_э} = 14$ atm. Hull plating is steel with a yield point $\sigma_T = 3000$ atm.

a. Shell thickness from Equation [22],

$$t = 1.4\,\frac{q_{1э}r}{\sigma_T} = 1.4\,\frac{14 \times 200}{3000} = 1.3 \text{ cm} = 13 \text{ mm}$$

b. Correction factor from Equation [24],

$$\eta_1 = \left(1 - \frac{0.3}{t}\right) = \left(1 - \frac{0.3}{1.3}\right) = 0.77$$

c. Correction factor η_2 from Table 10 for the adjusted stress $0.8\sigma_T = 0.8\ (3000) = 2400$ atm.,

$$\eta_2 = 0.8$$

d. Theoretical critical pressure from Equation [23],

$$q_э = \frac{1}{\eta_1\eta_2}\, q_{1э} = \frac{1}{0.77 \times 0.8}\ 14 = 22.7 \text{ atm}$$

e. From Table 8 or the graph in Figure 9, the quantity γ, corresponding to $q_э = 22.7$ and $\theta = t/r = 1.3/200 = 6.5 \times 10^{-3}$, is

$$\gamma = 2.88$$

f. The required frame spacing by Equation [25] is

$$l = \frac{r}{\gamma} = \frac{200}{2.88} = 70 \text{ cm}$$

Example 2. Find the shell thickness and frame spacing for the same shell radius $r = 200$ cm. and critical pressure $q_{1_э} = 14$ atm., for an increase in elastic limit σ_T to 4000 atm.

a. Shell thickness from Equation [22],

$$t = 1.4\,\frac{14 \times 200}{4000} = 0.98 \text{ cm} = 9.8 \text{ mm}$$

b. The correction factor η_1, by Equation [24],

$$\eta_1 = \left(1 - \frac{0.3}{0.98}\right) = 0.69$$

c. The correction factor η_2 from Table 10 for a stress of $0.8\,\sigma_T = 0.8\,(4000) = 3200$ atm.,

$$\eta_2 = 0.76$$

d. Theoretical critical pressure according to Equation [23],

$$q_э = \frac{1}{0.69 \times 0.76} \times 14 = 26.8 \text{ atm}$$

e. From Table 8 or the graph in Figure 9, the quantity γ corresponding to $q_э = 26.8$ and $\beta = 0.98/200 = 4.9 \times 10^{-3}$, is

$$\gamma = 6.4$$

f. Frame spacing from Equation [25],

$$l = \frac{200}{6.4} = 31 \text{ cm}$$

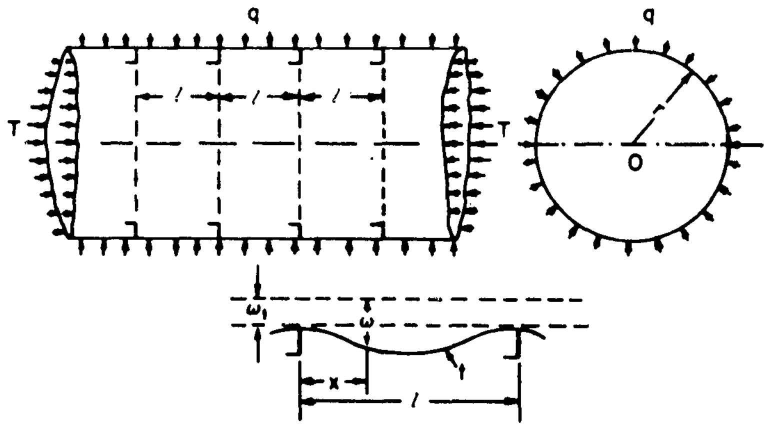

Figure 6

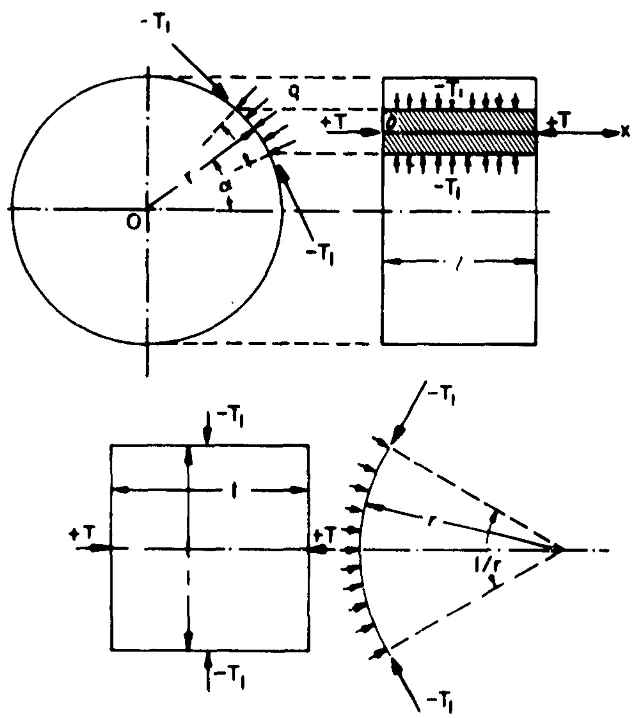

Figure 7

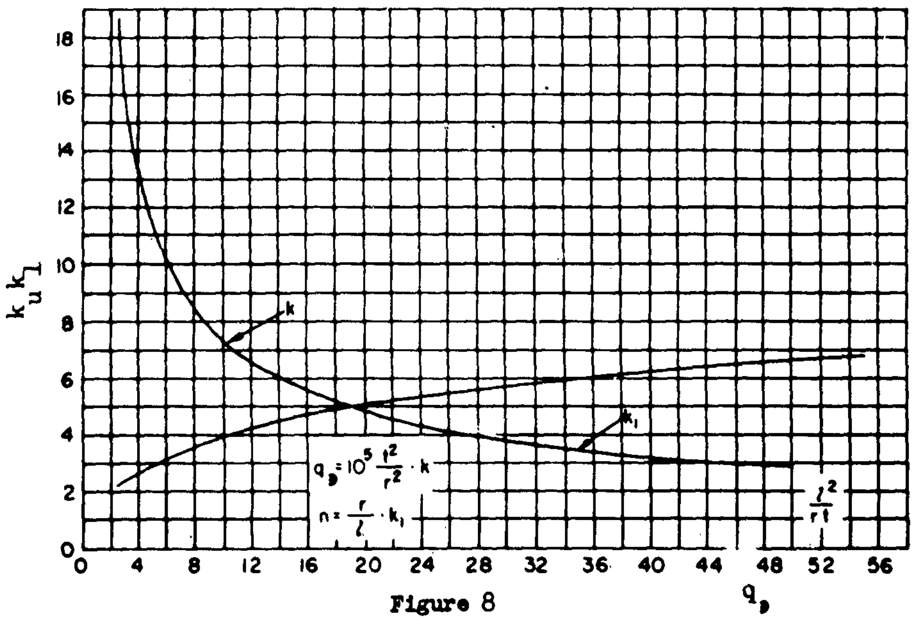

Figure 8

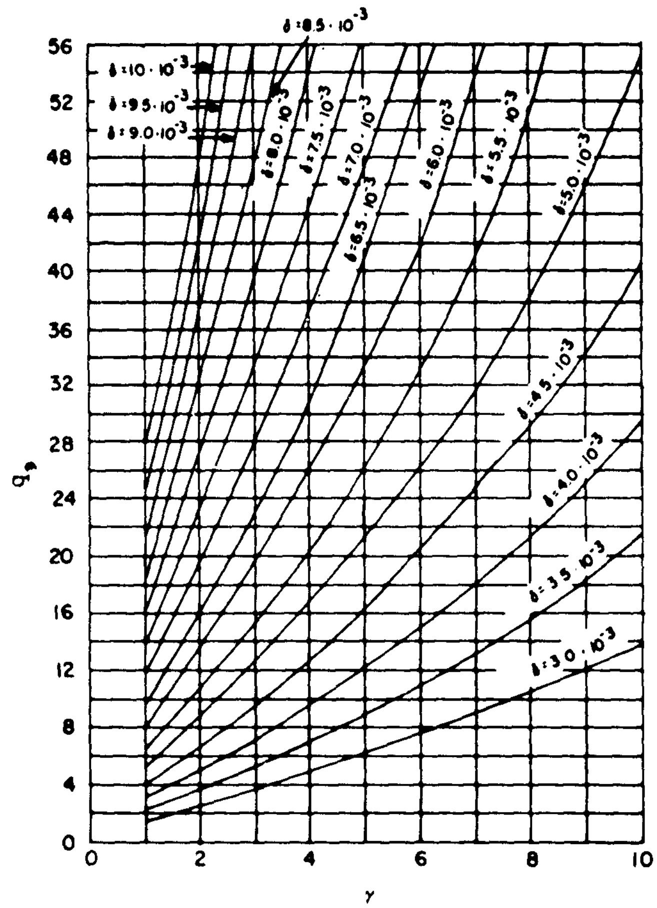

Figure 9

TABLE 1

Evaluation of Stresses and Deflections of the Shell

Name of Object (Item)		
Radius of the null $r = 200$ cm		Frame cross section area $F = 24.9\ cm^2$
Thickness of the shell $t = 1.3$ cm		Cross section area of the shell $lt = 91.0\ cm^2$
Frame spacing $l = 70$ cm		Total area of the cross section $F + lt = 115.9\ cm^2$
$\delta = \frac{t}{r} = 6.5 \cdot 10^{-3}$	$\sqrt{100}\ \delta = 0.805$	$f = 0.85 \frac{lt}{F} = 3.1$
$\gamma = \frac{r}{t} = 2.86$	$\gamma^2 = 8.15$	$k = \frac{F}{F + lt} = 0.215$
$q = 15$ am	$E = 2.10^6$ am	$\sqrt[3]{\frac{q}{\delta_\gamma}} = 9.31$
$a = 3.46 \frac{\sqrt{tr}}{l} = 0.795$		$c = 0.062 \sqrt[3]{\frac{q}{\delta_\gamma}} : \sqrt{100\delta} = 0.715$
$N = 1.025$		$P = 2.44$
$M = 0.863$		$Q = -0.400$

Calculation of deflections			Calculation of stresses					
1	$2\ k\ N$	0.440	12		Reaction of the shell on the frame R		$\frac{EF}{r^2} \cdot (10)$	134
2	$x = \frac{k}{1 + (1)}$	0.149	13		Stress in the cross section of the frame		$-\frac{E}{r} \cdot (10)$	-1070
3	$37\ \gamma^2 q \cdot (2)$	674	14	On the supporting contour	Transverse cross section σ_1	Inner surface	$-(6) - (4)$	-2795
4	$P \cdot (3)$	1640	15	On the supporting contour	Transverse cross section σ_1	Outer surface	$-(6) + (4)$	485
5	$Q \cdot (3)$	-270	16	On the supporting contour	Longitudinal cross section σ_2	Inner surface	$0.3 \cdot (14) + (13)$	-1905
6	$\frac{q}{2\delta}$	1155	17	On the supporting contour	Longitudinal cross section σ_2	Outer surface	$0.3 \cdot (15) + (13)$	- 925
7	$3.4 \cdot M$	2.93	18	In the mead span	Transverse cross section σ_1	Inner surface	$-(6) - (5)$	- 885
8	$f + (7)$	6.03	19	In the mead span	Transverse cross section σ_1	Outer surface	$-(6) + (5)$	-1425
9	$\frac{r}{E\delta} q \cdot (2)$	0.0344	20	In the mead span	Longitudinal cross section σ_2	Inner surface	$\frac{f_{max}}{f_0} \cdot (13) + 0.3\ (18)$	-2345
10	Deflection of the frame $f_0 = f \cdot (9)$	0.107	21	In the mead span	Longitudinal cross section σ_2	Outer surface	$\frac{f_{max}}{f_0} \cdot (13) + 0.3\ (19)$	-2505
11	Deflection f_{max} $(8) \cdot (9)$	0.208	22		Mean stress in the shell σ_3		$\frac{f_{max}}{f_0} \cdot (13) + 0.3 \cdot (6)$	-2425

TABLE 2

Magnitude of Quantity N

a \ c	0	0.6	0.7	0.8	0.9
0.325	2.910	2.943	2.983	3.063	3.167
0.350	2.663	2.686	2.724	2.783	2.863
0.375	2.453	2.477	2.510	2.550	2.630
0.400	2.270	2.300	2.330	2.376	2.476
0.425	2.110	2.140	2.173	2.230	2.367
0.450	1.963	1.990	2.033	2.107	2.290
0.475	1.834	1.867	1.910	2.000	2.213
0.500	1.723	1.757	1.806	1.917	2.157
0.550	1.527	1.567	1.640	1.773	2.060
0.600	1.360	1.423	1.462	1.526	1.605
0.650	1.215	1.279	1.322	1.390	1.480
0.700	1.086	1.156	1.200	1.268	1.363
0.750	0.982	1.054	1.098	1.165	1.259
0.800	0.888	0.957	1.006	1.073	1.170
0.850	0.800	0.870	0.918	0.988	1.087
0.900	0.720	0.789	0.839	0.908	1.018
0.950	0.646	0.713	0.761	0.831	0.951
1.0	0.580	0.644	0.689	0.759	0.881
1.1	0.464	0.520	0.560	0.626	0.737
1.2	0.363	0.412	0.444	0.500	0.593
1.3	0.285	0.323	0.350	0.393	0.474
1.4	0.228	0.256	0.277	0.313	0.376
1.5	0.181	0.206	0.222	0.246	0.291
1.6	0.145	0.164	0.176	0.195	0.232
1.7	0.117	0.131	0.141	0.156	0.185
1.8	0.0940	0.1048	0.1124	0.128	0.167
1.9	0.0788	0.0874	0.0934	0.1045	0.1285
2.0	0.0652	0.0720	0.0680	0.0844	0.1060
2.1	0.0536	0.0586	0.0624	0.0672	0.0780
2.2	0.0434	0.0478	0.0508	0.0546	0.0622
2.3	0.0344	0.0404	0.0428	0.0456	0.0518
2.4	0.0312	0.0344	0.0364	0.0386	0.0442
2.5	0.0268	0.0292	0.0308	0.0326	0.0376
2.6	0.0232	0.0242	0.0260	0.0274	0.0316
2.7	0.0196	0.0210	0.0220	0.0236	0.0267
2.8	0.0170	0.0182	0.0188	0.0202	0.0230
2.9	0.0148	0.0158	0.0164	0.0178	0.0196
3.0	0.0130	0.0140	0.0144	0.0156	0.0170
3.1	0.0112	0.0120	0.0126	0.0136	0.0150
3.2	0.0102	0.0108	0.0112	0.0120	0.0130

TABLE 3

Magnitude of Quantity M

a \ c	00	0.6	0.7	0.8	0.9
0.325	1.700	1.720	1.740	1.766	1.795
0.350	1.580	1.600	1.635	1.665	1.695
0.375	1.473	1.495	1.535	1.565	1.600
0.400	1.386	1.410	1.440	1.470	1.507
0.425	1.310	1.335	1.357	1.387	1.425
0.450	1.243	1.270	1.290	1.317	1.355
0.475	1.187	1.216	1.240	1.265	1.305
0.500	1.140	1.170	1.195	1.222	1.262
0.550	1.058	1.093	1.117	1.147	1.195
0.600	0.990	1.033	1.063	1.100	1.153
0.650	0.925	0.976	1.011	1.054	1.115
0.700	0.863	0.919	0.958	1.008	1.077
0.750	0.803	0.862	0.905	0.960	1.040
0.800	0.743	0.805	0.849	0.909	1.000
0.850	0.684	0.747	0.792	0.855	0.958
0.900	0.626	0.689	0.735	0.800	0.914
0.950	0.570	0.632	0.677	0.744	0.860
1.000	0.516	0.575	0.620	0.687	0.802
1.1	0.416	0.472	0.510	0.575	0.684
1.2	0.334	0.383	0.410	0.464	0.560
1.3	0.264	0.302	0.326	0.370	0.450
1.4	0.210	0.240	0.260	0.297	0.356
1.5	0.168	0.191	0.207	0.234	0.280
1.6	0.134	0.152	0.165	0.183	0.222
1.7	0.109	0.122	0.132	0.145	0.178
1.8	0.0884	0.0998	0.172	0.1185	0.136
1.9	0.0738	0.0826	0.0888	0.0974	0.1108
2.0	0.0612	0.0676	0.0728	0.0792	0.0896
2.1	0.0500	0.0564	0.0592	0.0644	0.0726
2.2	0.0412	0.0458	0.0488	0.0528	0.0598
2.3	0.0344	0.0384	0.0406	0.0438	0.0496
2.4	0.0292	0.0320	0.0338	0.0364	0.0408
2.5	0.0250	0.0272	0.0286	0.0304	0.0340
2.6	0.0216	0.0232	0.0244	0.0260	0.0288
2.7	0.0188	0.0198	0.0208	0.0224	0.0248
2.8	0.0160	0.0170	0.0178	0.0194	0.0215
2.9	0.0138	0.0144	0.0152	0.0170	0.0186
3.0	0.0120	0.0126	0.0132	0.0148	0.0162
3.1	0.0106	0.0112	0.0118	0.0130	0.0140
3.2	0.0098	0.0101	0.0104	0.0116	0.0123

TABLE 4

Magnitude of Quantity P

a \ c	0	0.6	0.7	0.8	0.9
0.325	32.7	33.3	33.5	34.0	34.7
0.350	26.4	26.9	27.3	27.6	28.3
0.375	21.1	21.5	21.9	22.3	22.8
0.400	17.2	17.6	17.8	18.2	18.6
0.425	14.4	14.7	14.9	15.2	15.5
0.450	12.2	12.4	12.6	12.9	13.2
0.475	10.4	10.55	10.75	11.0	11.3
0.500	8.8	9.0	9.25	9.45	9.85
0.550	6.45	6.7	6.85	7.00	7.40
0.600	5.13	5.28	5.34	5.43	5.76
0.650	4.03	4.17	4.24	4.35	4.59
0.700	3.24	3.38	3.46	3.56	3.73
0.750	2.65	2.77	2.85	2.95	3.10
0.800	2.18	2.30	2.38	2.47	2.60
0.850	1.83	1.93	1.99	2.10	2.25
0.900	1.55	1.64	1.70	1.80	1.94
0.950	1.32	1.39	1.46	1.55	1.67
0.1	1.13	1.205	1.5	1.34	1.48
1.1	0.824	0.912	0.950	1.26	1.12
1.2	0.633	0.684	0.723	0.784	0.874
1.3	0.480	0.520	0.550	0.605	0.690
1.4	0.374	0.400	0.430	0.465	0.535
1.5	0.292	0.312	0.330	0.360	0.414
1.6	0.228	0.248	0.262	0.286	0.326
1.7	0.186	0.200	0.211	0.228	0.250
1.8	0.1490	0.1606	0.1694	0.1828	0.2048
1.9	0.1230	0.1324	0.1392	0.1472	0.1588
2.0	0.1004	0.1078	0.1134	0.1190	0.1270
2.1	0.0804	0.0876	0.0918	0.0976	0.1050
2.2	0.0654	0.0716	0.0754	0.0804	0.0870
2.3	0.0550	0.0594	0.0628	0.0666	0.0724
2.5	0.0404	0.0426	0.0444	0.0468	0.0500
2.6	0.0348	0.0370	0.0380	0.0400	0.0432
2.7	0.0302	0.0320	0.0330	0.0346	0.0384
2.8	0.0264	0.0278	0.0288	0.0302	0.0324
2.9	0.0232	0.0242	0.0252	0.0264	0.0280
3.0	0.0200	0.0210	0.0220	0.0232	0.0242
3.1	0.0174	0.0182	0.0190	0.0200	0.0212
3.2	0.0159	0.0160	0.0165	0.0175	0.0185

TABLE 5

Magnitude of Quantity Q

a \ c	0	0.6	0.7	0.8	0.9
0.325	0.156	150	0.150	0.138	0.130
0.350	0.194	0.210	0.223	0.223	0.223
0.375	0.225	0.257	0.273	0.287	0.295
0.400	0.248	0.288	0.304	0.330	0.352
0.425	0.252	0.298	0.321	0.353	0.390
0.450	0.248	0.284	0.320	0.350	0.400
0.475	0.218	0.255	0.294	0.329	0.390
0.500	0.177	0.177	0.218	0.290	0.368
0.550	0.048	0.095	0.125	0.165	0.228
0.600	−0.074	−0.060	−0.035	−0.007	0.045
0.650	−0.172	−0.170	−0.168	−0.160	−0.0140
0.700	−0.263	−0.254	−0.265	−0.278	−0.0294
0.750	−0.312	−0.332	−0.344	−0.373	−0.406
0.800	−0.346	−0.378	−0.401	−0.436	−0.490
0.850	−0.362	−0.404	−0.432	−0.480	−0.552
0.900	−0.360	−0.408	−0.442	−0.496	−0.582
0.950	−0.350	−0.400	−0.436	−0.494	−0.588
1.0	−0.334	−0.383	−0.420	−0.480	−0.582
1.1	−0.280	−0.339	−0.377	−0.430	−0.532
1.2	−0.232	−0.286	−0.312	−0.361	−0.448
1.3	−0.195	−0.228	−0.253	−0.294	−0.359
1.4	−0.160	−0.188	−0.208	−0.240	−0.296
1.5	−0.131	−0.151	−0.167	−0.192	−0.236
1.6	−0.105	−0.122	−0.133	−0.153	−0.186
1.7	−0.085	−0.099	−0.109	−0.123	−0.148
1.8	−0.068	−0.080	−0.088	−0.098	−0.115
1.9	−0.0564	−0.0656	−0.0732	−0.0826	−0.0976
2.0	−0.0476	−0.0548	−0.0598	−0.0676	−0.0788
2.1	−0.0408	−0.0458	−0.0496	−0.0552	−0.0624
2.2	−0.0346	−0.0378	−0.0408	−0.0446	−0.0510
2.3	−0.0292	−0.0310	−0.0330	−0.0362	−0.0424
2.4	−0.0244	−0.0258	−0.0374	−0.0300	−0.0352
2.5	−0.0210	−0.0220	−0.0232	−0.0254	−0.0292
2.6	−0.0181	−0.0191	−0.0200	−0.0216	−0.0248
2.7	−0.0158	−0.0168	−0.0176	−0.0190	−0.0214
2.8	−0.0136	−0.0144	−0.0152	−0.0167	−0.0190
2.9	−0.0116	−0.0124	−0.0132	−0.0144	−0.0168
3.0	−0.0100	−0.0108	−0.0114	−0.0126	−0.0148
3.1	−0.0080	−0.0088	−0.0094	−0.0108	−0.0128
3.2	−0.0075	−0.0080	−0.00845	−0.0092	−0.0102

TABLE 6

u	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.00
K	3.31	2.99	2.67	2.36	2.06	1.78	1.62	1.30	1.10	0.37	0.83
k	3.18	3.04	2.90	2.73	2.64	2.54	2.49	2.49	2.53	2.65	2.74

TABLE 7

Number of Waves n, Corresponding to the Loss of Stability of the Shell

Ranges of the change of the quantity $\frac{\gamma}{\delta} = \frac{r^2}{lt}$	Number of Waves n	Ranges of the change of the quantity $\frac{\gamma}{\delta} = \frac{r^2}{lt}$	Number of Waves n
0-- 1	2	285-- 345	14
1-- 4	3	345-- 435	15
4-- 10	4	435-- 535	16
10-- 17	5	535-- 645	17
17-- 28	6	645-- 900	18
28-- 45	7	900-- 1350	19
45-- 68	8	1350-- 1950	20
68-- 95	9	1950-- 3650	21
95--130	10	3650-- 7100	22
130--175	11	7100--15500	23
175--225	12	15500--31500	24
225--285	13	31500--63100	25

TABLE 8

Loading q_{ϕ}, Corresponding to the Loss of Stability of the Shell

($E = 2 \times 10^6$ kg/cm^2)

$\phi \cdot 10^{-3}$ / γ	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
1.0	1.6	2.3	3.1	4.1	5.3	6.5	8.0	9.7	11.7	14.0	16.2	18.6	21.3	24.3	27.5
1.5	2.0	2.9	4.1	5.5	7.0	8.9	11.0	12.9	15.5	19.0	22.0	25.7	29.2	33.5	37.4
2.0	2.5	3.7	5.3	6.8	9.0	11.0	14.0	17.6	19.4	24.1	27.6	32.3	37.2	41.7	47.3
2.5	3.1	4.4	6.1	8.2	10.7	13.5	16.5	20.0	23.4	28.0	33.0	39.5	46.0	53.5	
3.0	3.6	5.2	7.2	9.5	12.9	15.5	19.8	23.5	28.0	33.6	40.0	47.5	55.0		
3.5	4.3	6.3	8.3	11.0	14.8	17.4	21.3	26.2	32.5	38.0	46.0	55.0			
4.0	4.9	7.0	9.5	12.6	17.0	20.5	25.5	31.0	37.5	44.0	53.5				
4.5	5.5	8.0	10.8	14.5	19.0	23.0	27.5	32.5	42.5	49.5					
5.0	6.1	9.2	12.3	16.4	21.4	26.0	33.5	41.0	49.5						
5.5	6.8	9.8	13.5	18.5	23.8	29.8	37.5	46.2							
6.0	7.5	11.0	15.0	20.5	26.3	33.2	41.5	52.8							
6.5	8.2	12.0	16.5	22.6	28.8	37.0	47.3								
7.0	9.0	13.0	18.1	24.8	31.7	41.0	54.0								
7.5	9.8	14.2	19.8	27.0	35.0	46.2									
8.0	10.5	15.4	21.5	29.2	37.5	51.7									
8.5	11.3	16.5	23.4	31.5	42.2										
9.0	12.0	18.2	25.3	34.0	46.2										
9.5	12.8	20.0	27.3	37.0	50.0										
10.0	13.7	21.5	29.5	40.5	54.7										

TABLE 9

Correction Factor η_1

t mm / Formulae	4	5	6	7	8	9	10	11	12	13	15
(15)	0.45	0.57	0.69	0.80	0.90	0.99	1.05	1.10	1.15	1.18	1.21
(16)	0.42	0.52	0.62	0.72	0.81	0.88	0.93	0.97	1.00	1.02	1.04
(17)	0.30	0.39	0.48	0.55	0.62	0.68	0.72	0.76	0.78	0.80	0.83

TABLE 10

Corrected Euler's Stresses and Correction Factors

Euler's Stresses as per Theoretical Formulas	Corrected Euler's Stresses and Correction Factors					
	Ordinary steel		Steel of higher quality		Manganese steel	
$\sigma_э$	$\sigma_{1э}$	η_2	$\sigma_{1э}$	η_2	$\sigma_{1э}$	η_2
0	0	--	0	--	0	--
200	200	1.00	200	1.00	200	1.00
400	400	1.00	400	1.00	400	1.00
600	560	0.93	580	0.97	580	0.97
800	720	0.90	760	0.95	760	0.95
1000	880	0.88	940	0.94	940	0.94
1200	1020	0.85	1110	0.92	1120	0.92
1400	1160	0.83	1280	0.91	1300	0.91
1600	1280	0.80	1440	0.90	1460	0.91
1800	1390	0.78	1600	0.89	1630	0.90
2000	1480	0.74	1740	0.87	1800	0.90
2200	1570	0.71	1860	0.84	1940	0.88
2400	1640	0.68	2020	0.84	2100	0.87
2600	1720	0.66	2140	0.82	2240	0.86
2800	1780	0.64	2260	0.81	2380	0.85
3000	1840	0.62	2400	0.80	2500	0.83
3200	1890	0.59	2500	0.78	2640	0.82
3400	1940	0.57	2600	0.76	2760	0.81
3600	1980	0.55	2660	0.74	2900	0.80
3800	2020	0.53	2710	0.71	3000	0.79
4000	2060	0.51	2780	0.69	3100	0.77
4200	2100	0.50	2860	0.68	3200	0.76
4400	2140	0.49	2900	0.66	3300	0.75
4600	2180	0.47	2960	0.64	3380	0.74
4800	2200	0.46	3000	0.62	3440	0.72
5000	2230	0.44	3060	0.61	3520	0.70
5200	2260	0.43	3100	0.60	3580	0.69
5400	2290	0.42	3200	0.59	3640	0.67
5600	2320	0.41	3240	0.58	3720	0.66
5800	2340	0.40	3280	0.57	3780	0.65
6000	2360	6.39	3320	0.55	3820	0.64
6500	2420	0.37	3360	0.52	3940	0.60
7000	2460	0.35	3440	0.49	4040	0.57
7500	2500	0.33	3520	0.47	4140	0.55
8000	2540	0.32	3580	0.45	4240	0.53
8500	2580	0.30	3640	0.42	4380	0.51
9000	2600	0.29	3700	0.41	4420	0.49
9500	2640	0.28	3740	0.39	4500	0.47
10000	2670	0.27	3820	0.38	4580	0.46

TABLE 11

Calculation of the Stability of the Shell According to Formulae (15), (16), and (17). Actual Critical Pressure q_{1_b} and Average Compressive Stress σ_{1_b}, in Metric Atmospheres.

	$t_1 = 1.3\ cm$	$r = 200\ cm$	$l = 70\ cm$
	$t = 1.25\ cm$	$\gamma = \frac{r}{l} = 2.86$	$\delta = \frac{t}{r} = 6.25 \cdot 10^{-3}$
By formula (17)	$n = 16$	$\alpha^2 = \pi^2\gamma^2 = 80.7$	$x = 0.149$; $\eta_1 = 0.79$
By formula (17)	$q_b = \frac{4.4 \cdot 10^6 \cdot \delta}{2(1 - 0.85x)\, n^2 + \alpha^2} \left[\frac{\alpha^2}{3n^2 + \alpha^2} + \frac{\delta^2}{12} (n^2 + \alpha^2)^2 \right] = 24\ atm$		
By formula (17)	$\sigma_b = 1.1\eta_1 \frac{q_b r}{t} = 3340\ atm$	$\eta_2 = 0.77$	
By formula (17)	$q_{1_b} = \eta_2\eta_1 q_b = 14.6\ atm$	$\sigma_{1_b} = \eta_2\sigma_b = 2570\ atm$	
By formula (16)	$\delta_1 = \frac{100\, t}{r} = 0.625$	$\eta_1 = 1.01$	
By formula (16)	$q_b = 19\delta_1^2 (\gamma^2 \cdot \delta_1^2)^{0.58} = 19\ atm$		
By formula (16)	$\sigma_b = 1.1\eta_1 \frac{q_b r}{t} = 3380\ atm$	$\eta_2 = 0.77$	
By formula (16)	$q_{1_b} = \eta_1\eta_2 q_b = 14.8\ atm$	$\sigma_{1_b} = \eta_2\sigma_b = 2600\ atm$	
	$\delta_2 = \frac{100\, t}{l} = 1.79$	$\eta_1 = 1.165$	
By formula (15)	$q_b = 18.3\, \delta_1^{\frac{3}{2}}\, \delta_2 = 16.2\ atm$		
By formula (15)	$\sigma_b = 1.1\eta_1 \frac{q_b r}{t} = 3300\ atm$	$\eta_2 = 0.77$	
By formula (15)	$q_{1_b} = \eta_1\eta_2 q_b = 14.5\ atm$	$\sigma_{1_b} = \eta_2\sigma_b = 2540\ atm$	

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CHAPTER 3

STRENGTH ANALYSIS OF FRAMES.

8. CONTRIBUTION OF FRAMES IN SECURING THE STRENGTH OF THE HULL AND THEIR VARIOUS DESIGNS

1. In the design of the outer hull of submarines, frames play the same role as the ordinary system of transverse members in the hull of surface vessels; these frames assure, in essence, the local strength of the outer hull subjected to hydrostatic pressure, wave impact, and other loadings, depending on the disposition of the corresponding parts of this hull (side tanks, deck tanks, etc.). Therefore, the strength analysis of frames of outer hulls may be carried out by the usual methods and procedures applicable in surface-vessel construction, considering the practical requirements and structural peculiarities of submarine hulls.

As distinctive features of the frames of the outer hulls of submarines (compared with those of surface vessels of corresponding dimensions), we may enumerate their more complicated design and the necessity of their structural correlation with the framing of the pressure hull of the submarine. The requirements of such a correlation may make it expedient to completely or partially identify the frames of the outer hull with those of the pressure hull at the expense of their proper mounting and strengthening in accordance with those stability requirements which must be met by the pressure hulls of submarines.

The strength of frames of those parts of the outer hull serving as the walls of ballast side tanks, deck tanks, and other tanks, must be checked with respect to the loading changes occasioned by blowing down these tanks or other specific requirements.

2. The role of frames of the pressure hull in guaranteeing its general transverse strength at the limiting depth of submergence of the submarine is determined by the design of the frame structure, i.e. by the type of organization of the corresponding sections of the pressure hull. In this respect, a distinction must be made between circular and non-circular frames. From the point of view of structural mechanics, both represent closed, curved beams subjected to the action of distributed compressive loading in the form of reactive stresses set up between the frames and the plating of the pressure hull during bending of the plating, caused by the water pressure.

For the case of a purely circular hull, the plating itself almost completely balances out the pressure of the water uniformly distributed over its surface, being loaded by chain compressive forces and only partially transmitting this pressure to the frames in the form of the reactions mentioned, which appear as a result of the fact that the frames impede the free

compression of the plating at points of their mounting.

These reactions of the frames somewhat relieve the load on the plating only at the points where they are mounted, while not influencing noticeably the magnitude of stresses in the span between frames. Furthermore, occurrence of these reactions causes a bending of the plating in the spans between the frames; consequently, supplementary stresses occur in the supporting sections of the plating due to bending. The greater the rigidity of the frames, the greater is this bending. As the result of the foregoing (and as follows also from the formulas for calculation and from the numerical examples for the bending of the plating, cited in Part II, Chapter 1), the presence of frames not only does not diminish stresses in the plating but increases them appreciably, and in a larger measure with greater frame rigidity. This circumstance has stimulated several authors to recommend reducing the dimensions of the spaces adjacent to the bulkheads, to compensate thus for the effect on stresses in the plating resulting from high rigidity of the bulkheads. However, such a measure can not be considered justifiable, since the increase of these stresses occurs only close to the frame system which supports the plating, a fact that permits considering these stresses as purely local in character.

Despite their negative influence on the stress magnitude in the plating, circular frames appear to be an absolutely necessary element in the structure of a hull, since they play a decisive part in assuring the stability of the plating in the spans between the transverse bulkheads. The frames break up these spans into shorter ones so that the stability of the plating becomes sufficient to respond to and equilibrate the outside water pressure acting upon it. In addition to this fundamental purpose of circular frames, they must also react to those bending stresses which occur in the sections of circular hulls resulting from non-uniformly distributed external hydrostatic loading along the perimeter (Part 1, Section 2). However, due to their comparatively small magnitude, these bending stresses have practically no effect on the dimensions of circular frames required to satisfy their basic purpose as stated.

Non-circular frames (elliptical and others), in addition to guaranteeing the stability of the plating, must also assume the large bending stresses occurring in the hull sections whose shapes differ from a regular circle.

Design dimensions of non-circular frames, in contrast to circular ones, are determined with respect to their resistance to bending. This leads to a significant increase in dead weight in comparison to circular frames; this increase becomes very much greater with increase of the design depth of submergence of the hull. The latter circumstance appears to be the chief reason for refusal to use elliptical hull shapes on contemporary submarines, though elliptical hull sections were widely employed in the early period of submarine development when the limiting depth of submergence was

more moderate.

3. It is especially fitting to note the great significance of the frames in assuring the strength of the hull against underwater explosions and other local, incidental stresses to which it may be subjected under the various conditions of operation of a submarine (impact against the bottom and when mooring, cruising in ice, etc.).

The experience gained by combat operation of submarines has shown that the hydraulic impact of water during depth bomb attack, or resulting from mines or aerial bombs, often produced dents and hollows in the plating in the areas between the frames without causing leaks. Sections of the plating thus deformed are no longer able to transmit the water pressure upon them to adjacent undeformed portions of the structure; these areas of the plating will operate as flexible plates transferring the pressure of the water to the frames. In this case, as in all other cases of local impression or denting of the plating, the survival of the hull at great depths of immersion will be guaranteed only by the local strength of the frames.

The role of the frames mentioned in assuring combat strength of the hull, which has become particularly significant in view of the contemporary development of submarine warfare, makes the establishment of the structural specifications of frames mandatory not only according to the calculation of their general stability for strength requirements, but must take into account also the necessity for a sufficient local stability -- these features being determined by previous practical combat operation of submarines as a general guide.

9. STRENGTH ANALYSIS OF CIRCULAR PRISMATIC FRAMES.

1. **Stability Analysis.** Circular frames must remain stable at a loading on the hull, corresponding to the instant of loss of stability of the plating in the spans between the frames, i.e., at a loading q_p corresponding to the design depth of submergence of the submarine.

Assuming that every frame with the belt of plating adjoining it operates individually, i.e., disregarding the favorable effect on the frame stability of the presence of transverse bulkheads, the conditions cited above require satisfaction of the following inequality (Part 2, Section 31):

$$\frac{3EI}{r^3} > q_p l \qquad [1]$$

where q_p is the design pressure equal to 0.1 h_p (h_p = the design depth of submergence in meters); l is the distance between the frames; I is the moment of inertia of the sectional area of the frame, including the sectional area of the adjoining portion of the plating lt; r is the radius of the circle of a frame, calculated to the outer edge of the plating (for greater exactitude in calculation, we should substitute for r^3 in Equation [1] the quantity $r_1^2 r$, where r_1 is the radius of the frame to the neutral

axis).

The left term of the inequality [1] represents the well-known expression for the Euler load of a ring subjected to a uniformly distributed circumferential load. A section of this ring is taken as consisting of the section of the frame itself and the section of that portion of the plating adjoining it, $l t$. In reality, the plating, as a result of loss of stability in the span between the frames, will no longer be able to assume a full share of the bending load in the ring under consideration; therefore, the left side of inequality [1] may prove to be too large. A certain increase must also result from the absence in inequality [1] of a correction factor, less than unity, required to account for the effect of the stress magnitude on stability (Part 2, Section 33).

These circumstances are practically taken into account simultaneously with favorable effect on the stability of frames of transverse bulkheads by introducing, into the expression for Euler's loading of a frame, a practical coefficient by which the structural dimensions of frames, which satisfy Equation [1], are found to be in accord with previous experience in submarine operation insofar as survival of the hull be concerned (Part 1, Section 8, Paragraph 3).

Taking this practical coefficient as equal to 1.5, we get the following formulas for the analysis of the stability of circular frames (in kg, cm).

The magnitude of the design load q_p, corresponding to the given magnitudes of r, l and I, will be

$$q_p \leq \frac{3\ EI}{1.5\ r^3 l}$$

or

$$q_p \leq \frac{2EI}{r^3 l}\ \text{atm.} \qquad [2]$$

The necessary magnitude of the moment of inertia of the cross-sectional area of the frame (together with the plating) for given quantities q_p, r and l, is

$$I \geq \frac{1.5\, r^3 l}{3E} q_p$$

that is, [3]

$$I \geq \frac{r^3 l}{2E} q_p\ [\text{cm}^4]$$

It must be emphasized that the design Formula [3] which determines the structural dimensions of frames, must guarantee not only the general

stability but also the local stability and strength upon which the extent of combat strength and survival of submarine hulls depends. Therefore, any decrease of the structural dimensions of frames contrary to those required by Equation [3] is impossible without deleterious effect to the combat strength of the hull, even though such reduction might be justified from the point of view of guaranteeing the general stability of the frames (inclusion of supplementary structural connections such as tank walls, decks, bulkheads, etc.).

2. Stress Analysis. A circular frame, from the point of view of structural mechanics, is considered to be a prismatic ring, subjected to the action of a load uniformly distributed along its circumference. In the cross sections of such a ring, as is well-known, bending moments and shearing forces are absent and only longitudinal forces are set up; these are equal to

$$S = pr \qquad [4]$$

where p is the external load, per unit of length of the ring; r is the radius of the circle along which the load is applied.

The stress in the sections of the ring, corresponding to the loading p, is found to be

$$\sigma = \frac{S}{F} = \frac{pr}{F} \qquad [5]$$

where F is the sectional area of the ring.

The frames of the hull are rings of molded (profiled) steel, welded or riveted to the hull plating. When a loading p applies on the frames, a stress develops between them and the plating. In calculation of the plating, this stress was designated by the symbol R and was defined by the expression (Part 1, Section 5, Paragraph 3):

$$R = \frac{EF}{r^2}\,\omega_0 \qquad [6]$$

where ω_0 is the compression of supporting sections of plating (of frames) at a given design load q_p, applied on the hull of a submarine.

Taking p = R in Equations [4] and [5], we get the following expressions for the longitudinal forces and stresses in the sections of the frame:

$$S = EF\,\frac{\omega_0}{r} \qquad [7]$$

$$\sigma = E\,\frac{\omega_0}{r} \qquad [8]$$

In reality, the stresses in the sections of a frame may exceed significantly the theoretical stress, determined by Equation [8], as the result

of bending moments occurring in the section of the frame. Bending of frames arises from (a) the compensation, on the frames, of the forces of buoyancy and of the dead weight of a submarine (Part 1, Section 2) and (b) the consequence of a certain and practically unavoidable deviation of the shape of the frames from the true circular form (Part 1, Section 23). Additionally, it must be considered that after stability has been lost in the plating, in the spans between frames, the load on the frames must increase sharply, since the plating is no longer able to react to the water pressure applied on it.

All these circumstances force to limit the magnitude of the stresses obtained by the theoretical Formula [8], in order to guarantee the absence of excessive stresses in the sections of the frames which may greatly weaken their stability.

Calculations based on current industrial tolerance for determination of permissible deviation from true circular design of frames, led to the conclusion that stresses in the sections of frames calculated according to Equation [8] must not exceed 35% of yield stresses of the frame material.

In this case, the formula for checking the stability of the frames with respect to stress is stated:

$$\frac{E\,\omega_0}{r} \leq 0.35\,\sigma_T \qquad [9]$$

where r is the outside radius of the frame, σ_T is the yield stress of the material, ω_0 is the compression of the frames resulting from the application on the hull of the design load q_p.

The stresses in the sections of a frame, determined by the left side of Equation [9], are obtained in the calculation outlined in Chapter II for the strength of plating and may, therefore, be taken directly from that analysis; see Table 1.

3. Tabulated Calculation of the Stability of a Frame. The check calculations of the stability of frames according to Equations [2] and [9] should be tabulated; see Table 12.

In the upper part of the table are given the radius of the hull r, the thickness of the plating t, the length of the span l, the profile of the frame, sectional area F, the distance of the center of gravity of its area from the lower edge e (position of neutral axis), and the moment of inertia of its area i. Further, the usual table for computing the moment of inertia of a frame section is made up, consisting of the assumed profile and a portion of the plating of a width equal to the length of the span. In the next to the last line the load* is computed, determined by the

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*Editor's Note: The author calls it "critical pressure". See Table 1, q = 15 atm.

right-hand term of Equation [2], which is compared with the design load q_p.

In the last line is written the stress in the frame due to the design load; this stress can be taken according to line 13 of Table 1. This stress, is compared with the standard, established by Equation [9].

Table 12 contains initial data which were examined in previous numerical examples.

10. STRENGTH ANALYSIS OF CIRCULAR NON-PRISMATIC FRAMES.*

By the term non-prismatic frames is understood a circular frame consisting of individual parts of different cross section, whereby the neutral axes of the cross sections of these parts may lie on circles of different radius. Such frames in practical construction of submarine hulls are encountered in the following forms.

1. The neutral axis of the entire frame forms one circle, but the moments of inertia (rigidity) of the sections of the frames vary along the length of this circumference (Figure 93).

Investigation of the deformation of such a frame leads to the conclusion that a change of rigidity (i.e. of moments of inertia) of the sections of a frame, loaded at a uniform pressure, does not cause supplementary deformations in it, independently of the radius of the circle along whose circumference the external pressure is applied.

2. The rigidity of the one part of the frame is large in comparison with the rigidity of the remaining portion of the frame whereby the neutral axes of these parts of the frame lie on circles of differing radii (Figure 94).

In this case, on the portion of the ring which is less rigid, no supplementary stresses occur, but in the more rigid part there appears a constant bending moment, given by

$$M = qr(r_1 - r_2) \qquad [10]$$

where q is the uniform pressure on the frame; r is the radius of the circle upon which the pressure applies; r_1 is the radius of the neutral axis of the more rigid part of the frame; r_2 is the radius of the neutral axis of the less rigid part of the frame.

3. The frame consists of two half-rings of equal rigidity whose neutral axes are equidistant by a distance e from the middle circle at whose periphery the external pressure applies (Figure 96). In this case, in the sections of the frame, there are produced the following moments and forces:

bending moments:

$$M_\alpha = qre\left(1 - \frac{4}{\pi}\sin\alpha\right) \qquad [11]$$

*Theoretical foundations cited in Sections 25 and 26, of Part 2.

axial forces:

$$S_\alpha = -qr\left(1 - \frac{4}{\pi}\beta \sin \alpha\right) \qquad [12]$$

shearing forces:

$$T_\alpha = \frac{4}{\pi} q e \cos \alpha \qquad [13]$$

where r is the radius of the circle at which the external pressure acts;

e is the distance from the neutral axes of both parts of the frame to the circumference of the circle of radius r;

α is the angle defining the position of the section of the frame;

$\beta = e/r$ is a small quantity whose square may be disregarded in comparison with unity.

4. The frame has a local increase of thickness (height) whereby the external pressure is applied at the outer circumference of the frame. [Figure 95].

In this case, supplementary bending moments occur only in the strengthened portion of the frame; their magnitude is determined by Equation [10].

11. STRENGTH ANALYSIS OF ELLIPTICAL FRAMES.

1. In the design of submarines, elliptical frames represent from the point of view of structural mechanics elastic rings of elliptical shape subjected to the action of an external distributed pressure, transmitted to them through the plating which is secured to them and to terminal rigid bulkheads of the same elliptical shape. Investigation of the deformation of such a complex system encounters such great difficulties that a solution of this problem, suitable for practical application, does not exist. Therefore, for calculating the stability of elliptical frames, approximations, and consequently provisional methods of computation must be used, correcting these values by the establishment of appropriate standards for the allowable stresses, correlated with previous practice in submarine construction.

2. Let us first investigate the deformation of an isolated elliptical frame under the action of a load uniformly distributed along its circumference (Figure 10). Such a frame may be considered as circular, but having a comparatively large initial deflection, i.e. a large deviation from its original true circular shape. In the sections of such a frame, in addition to the axial forces, occurring in the usual circular frame, there must also occur bending moments proportional in magnitude and sign to the total, i.e. initial and elastic, deflections of the frame.

Indicating by a and b the major and the minor semi-axes of the ellipse, it is possible, if their difference is small, to take the radius of the mean circle of the ellipse as equal to $r = 1/2(a + b)$ and the

deflection of the initial bending at the ends of the semi-axes as equal to

$$f_a = a - r = \frac{1}{2}(a - b); \qquad f_b = b - r = -\frac{1}{2}(a - b) \qquad [14]$$

Applying to this case the conclusions stated in Part 2, Section 29, for a ring having an initial deflection, we get:

elastic deflection (sag from true circle):

$$\Delta f_a = f_a \frac{1}{(p_э/p)-1}; \qquad \Delta f_b = f_b \frac{1}{(p_э/p)-1} \qquad [15]$$

total deflection (sag):

$$\left.\begin{aligned} f_a + \Delta f_a &= f_a \frac{1}{1-p/p_э} = f_a\mu \\ f_b + \Delta f_b &= f_b \frac{1}{1-p/p_э} = f_b\mu \end{aligned}\right\} \qquad [16]$$

bending moments without consideration of elastic deflection:

$$\left.\begin{aligned} M_a' &= p_a f_a = \frac{pa^2}{2}\left(1 - \frac{b}{a}\right) \\ M_b' &= p_b f_b = -\frac{pb^2}{2}\left(\frac{a}{b} - 1\right) \end{aligned}\right\} \qquad [17]$$

bending moments with consideration of elastic deflection:

$$\begin{aligned} M_a &= pa(f_a + \Delta f_a) = M_a'\mu \\ M_b &= pb(f_b + \Delta f_b) = M_b'\mu \end{aligned} \qquad [18]$$

where p is the load acting on the frame;

$p_э$ is the Euler load of the frame;

pa, pb, are the axial forces in the section of the frame.

From the expressions cited, it is evident that the effect of elastic deformation of an elliptical frame on the bending moments in its sections is taken into account by a factor

$$\mu = \frac{1}{1-p/p_э} \qquad [19]$$

At a loading p on an elliptical frame equal to half the Euler load, $p_э$, this factor is found equal to 2. At further increase of the load, the importance of this factor increases rapidly as shown in Figure 11.

From the foregoing, it is clear to what extent elliptical frames are unfavorable in comparison with purely circular ones, especially at loads closely approaching Euler's.

From the expressions obtained, it is likewise evident that, in the given case, proportionality between the magnitude of the applied load and the stresses produced by the load is absent in the sections of an elliptical frame. This circumstance is taken into account by the substitution in the calculations for the strength of submarine hulls of assumed safety factors; substituted not into the stresses but into the acting load.

The expressions given under Equation [17] for the quantities M_a' and M_b' were developed under the assumption of a small difference between the axes of an elliptical frame as is generally true for the elliptical frames of submarine pressure hulls.

More exact expressions for the quantities M_a' and M_b', developed without the limitation indicated above, are given below and were determined as a result of an analytical investigation of the deformation of an elliptical ring. For this purpose, an investigation of the approximate solution of the elliptical integrals encountered in such a problem was used.*

$$\left.\begin{aligned} M_a' &= \frac{pa^2}{4} e_0^2 \left(1 + \frac{e_0^2}{8}\right) \\ M_b' &= \frac{pa^2}{4} e_0^2 \left(1 - \frac{e_0^2}{8}\right) \end{aligned}\right\} \quad [20]$$

where $e_0^2 = 1 - b^2/a^2$, (e_0 = the eccentricity of the ellipse).**

In the presence of a rigid connecting member (stanchion or stay, bulkhead, deck) coincident with the major or minor axis of an elliptical frame, the greatest bending moments at the ends of the axes and the longitudinal forces in the rigid member are found to be equal to:

with connecting member coincident with major axis:

$$\left.\begin{aligned} M_a &= -0.105\, p a^2 e_0^2 \\ M_b &= -0.088\, p a^2 e_0^2 \\ S &= 1.1 \frac{pa^2}{b} e_0^2 \end{aligned}\right\} \quad [21]$$

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*Novozhilov, V. V., "Calculation of Stability of Elliptical Frames of Submarines", Collection, HNBK (NIVK), Number 4, 1935; Translator's Note: NIVK might be an abbreviation for "National Institute of War Ships."

**Editor's Note: The expression for M_b' is obviously in error. Copied as it stands in original.

with connecting member coincident with minor axis:

$$\left.\begin{aligned} M_b &= 0.1\,pa^2\epsilon_0^2 \\ M_a &= 0.088\,pa^2\epsilon_0^2 \\ S &= -1.1\,pa\epsilon_0^2 \end{aligned}\right\} \qquad [22]$$

3. In the determination of the magnitude of Euler's loading of a frame, which enters into the expression for the coefficient μ, it is appropriate to consider that the frames situated in the span between the transverse bulkheads of the hull can not bend individually and independently because of their connection to the plating. Moreover, the plating in turn (secured to the rigid bulkheads) resists such bending by developing shear forces in its middle surface. This favorable influence of transverse bulkheads on the bending of elliptical frames can be taken into account by application of the conclusions found in Part 2, Section 18.

Euler's load of individual frames, determined by the well-known expression,

$$p_э = \frac{3EI}{r^3} \qquad [23]$$

corresponds to a bending of frames in two waves (n = 2, Figure 56).

Euler's load for the entire assembly of frames between the transverse bulkheads under this type of bending can be determined by Equation [52] cited in Part 2, Section 18, assuming in this expression n = 2 and taking $p_э = q_э l$, where l is the distance between the frames, and where $q_э$ is Euler's load, referred to a unit surface of the hull plating.

Substituting in the indicated expression n = 2 and $q_э = p_э/l$, we get

$$p_э = l\,\frac{E\delta}{4\,\dfrac{9+\alpha^2}{12+\alpha^2}+\dfrac{1}{2}\alpha^2}\left[\frac{\alpha^2}{12+\alpha^2}+9\,\frac{I}{r^3 l\delta}\right] \qquad [24]$$

where

$$\left.\begin{aligned} \delta &= t/r \\ \alpha^2 &= \pi^2 \times \frac{r^2}{L^2} \end{aligned}\right\} \text{Conventional Symbols}$$

t is the thickness of the hull plating

r is the radius of the frames

L is the distance between transverse bulkheads

I is the moment of inertia of the sectional area of a frame, with consideration for the area of plating equal to lt

l is the distance between the frames.

12. STRENGTH ANALYSIS OF FRAMES OF ARBITRARY SHAPE.

Strength Analysis of frames of arbitrary shape, i.e. determination of stresses arising in the section of such a frame, is accomplished most simply by application of the method of potential energy (principle of least work).

Application of this method to the frame of arbitrary shape is illustrated in the following example, worked out by Prof. P. F. Papkovich.

As an example, let us examine a closed curvilinear frame of a two-hull submarine with a vertical axis of symmetry. At a great depth, the pressure of the water may be considered very close to uniform. It usually acts on the inner hull of the vessel (Figures 12 and 13). The dead weight of the individual parts of the hull and even of mechanisms in submarines, whose inner contour deviates from a circle, produces such small stresses compared to those which result from the water pressure that the effect of weight may usually be neglected.

In these cases, the frame ring of the vessel may be analyzed solely for external pressure, distributed uniformly over the surface of the ship's skin. Since it has a vertical axis of symmetry and is loaded symmetrically with respect to this axis, its deformation may be considered also symmetrical with respect to this axis.

This makes it possible to assume that in the sections coincident with the diametral plane, the shear force in the frame ring equals zero. The force and moments acting in these sections thus reduce only to the two horizontal forces R_1 and R_2 and the two couples $\mathfrak{M}_1$ and $\mathfrak{M}_2$, applied to the half ring in its upper and lower supporting sections.

Let these forces and moments act on the right-hand half ring in the direction of the arrows as shown in Figure 13. For their determination we have at our disposal the two static equations. One of these gives the sum of the reactions $R_1 + R_2$, and the other connects one of these reactions with the load on the ring and with the moments $\mathfrak{M}_1$ and $\mathfrak{M}_2$. Therefore, for a given frame, as statically indeterminate quantities, we must take either both bending moments $\mathfrak{M}_1$ and $\mathfrak{M}_2$ or one of the moments $\mathfrak{M}$ and one of the reactions R.

Taking, as statically indeterminate quantities, one of the forces and one of the moments, it is necessary at the very end of the calculation to find small differences of relatively large quantities in order to determine the magnitude of the bending moment in the various sections of the frame. Usually when employing such a choice of unknowns, efforts to establish more than one significant figure of the result are unsuccessful and even this may be in error of 50% to 100% when all the calculations are carried out to three decimal places.

Conditions are entirely different, if, as statically indeterminate quantities, we take the reactive moments $\mathfrak{M}_1$ and $\mathfrak{M}_2$ in the upper and lower sections. In this case, the small differences of large quantities

must be determined at the very outset of the calculation, before the errors become too highly cumulative.

Carrying out the computation in this case, similarly with three places, it is usually possible to obtain as correct the two first significant figures in the result. Therefore, upon finding the indeterminateness of frames of such type, it is always indicated to take both moments $\mathfrak{M}_1$ and $\mathfrak{M}_2$ as the statically indeterminate quantities. Let us examine what type of computation must be developed for the solution of the problem, given such a selection of basic unknowns.

Let us introduce the following notation [Figure 13]:

- p is the magnitude of water pressure on a linear unit of the circumference of the frame ($p = ql$, where q is the pressure of water and l is the distance between frames);
- s is the coordinate of the center of gravity of the frame section being considered, measured along the circumference of its neutral axis;
- y is the vertical coordinate of the same point, measured from the line of application of the reactions;
- a is the length of the chord connecting those points of the pressure hull in which it is intersected by the section being examined and the upper vertical section;
- b is the lever arm of the center of gravity of this chord with respect to the center of gravity of the section being examined.

Examining the moments and forces in the upper part of the frame, we can write for the bending moment in a given section:

$$M = \mathfrak{M}_1 + pab - R_1 y \qquad [25]$$

In this expression, $\mathfrak{M}_1$, p and R_1 do not change when we pass from one transverse section of the frame to another, but a, b and y do change. Let in the lower supporting section a = A and b = B, and let the distance between the centers of gravity of the upper and lower sections equal B + C.

The bending moment in the lower section is

$$\mathfrak{M}_2 = \mathfrak{M}_1 + pAB - R_1(B + C),$$

from which

$$R_1 = -\frac{\mathfrak{M}_2 - \mathfrak{M}_1}{B + C} + p\frac{AB}{B + C}$$

Substituting this expression for R_1 in the general expression for the bending moment, we get.

$$M = \mathfrak{M}_1 + \frac{\mathfrak{M}_2 - \mathfrak{M}_1}{B + C}\ y + p\left(ab - \frac{AB}{B + C}y\right)$$

The quantity $(ab - \frac{AB}{B+C} y)$ can be calculated for each section of the frame by the numerical data taken from the diagram. Therefore, it can be considered as some function f(s), depending on the coordinate of the section s. Let us assume that the function f(s) is defined with the aid of the quantity

$$f(s) = \left(ab - \frac{AB}{B+C} y\right) : A^2 \qquad [26]$$

Thereupon, the expression for the moment becomes:

$$M = \mathfrak{M}_1 \left(1 - \frac{y}{B+C}\right) + \mathfrak{M}_2 \frac{y}{B+C} + pA^2 f(s) \qquad [27]$$

If, after the indeterminate is established, the displacements neither from shearing forces nor from longitudinal forces in individual sections are taken into account, but only the displacements due to bending, as is usual, we can take for the general expression for potential energy

$$V = \frac{1}{2} \int_0^s \frac{M^2}{EI} ds$$

If we assume that in the frame, under zero water pressure, there are no initial stresses, we can use the equations

$$\frac{\partial V}{\partial \mathfrak{M}_1} = 0, \quad \frac{\partial V}{\partial \mathfrak{M}_2} = 0,$$

for the determination of the indeterminate -- or what amounts to the same thing, the equations:

$$\int_0^s \frac{M\left(1 - \frac{y}{B+C}\right)}{EI} ds = 0; \quad \int_0^s \frac{M\left(\frac{y}{B+C}\right)}{EI} ds = 0$$

These two equations are equivalent to the expressions

$$\int_0^s \frac{M}{EI} ds = 0; \quad \int_0^s \frac{M}{EI} \frac{y}{B+C} ds = 0,$$

wherein integration is extended over the entire half circumference of the frame (along the entire half of the frame of the right side*).

---

*Editor's Note: Right side here may also refer to the starboard hull wall of a ship.

Developing these two equations, we get

$$\mathfrak{M}_1 \int_0^s \frac{1 - \frac{y}{B+C}}{EI}\, ds + \mathfrak{M}_2 \int_0^s \frac{\frac{y}{B+C}}{EI}\, ds + \gamma l A^2 \int_0^s \frac{f(s)}{EI}\, ds = 0 \qquad [28]$$

$$\mathfrak{M}_1 \int_0^s \frac{\left(1 + \frac{y}{B+C}\right) \frac{y}{B+C}}{EI}\, ds + \mathfrak{M}_2 \int_0^s \frac{\left(\frac{y}{B+C}\right)^2}{EI}\, ds + \gamma l A^2 \int_0^s \frac{f(s) \frac{y}{B+C}}{EI}\, ds = 0 \qquad [29]$$

The integrals entering these equations can be calculated graphically or by the well-known principles of approximate methods, since any number of particular values of integrand functions can be computed according to the numerical data taken from the diagram. After they have been found, the further solution of the equation does not cause difficulties. When the moments $\mathfrak{M}_1$ and $\mathfrak{M}_2$ have been determined, the moments in all remaining sections are easily found according to Equation [27].

For finding the values of the longitudinal forces S and the shear forces N it is possible, having computed R_1 according to the following formula

$$R_1 = \gamma l \frac{AB}{B+C} + \frac{\mathfrak{M}_1 - \mathfrak{M}_2}{B+C} \qquad [30]$$

to construct force triangles graphically, projecting the equivalent force of the water pressure upon the chord and the force R_1 upon the tangent and normal to the section considered.

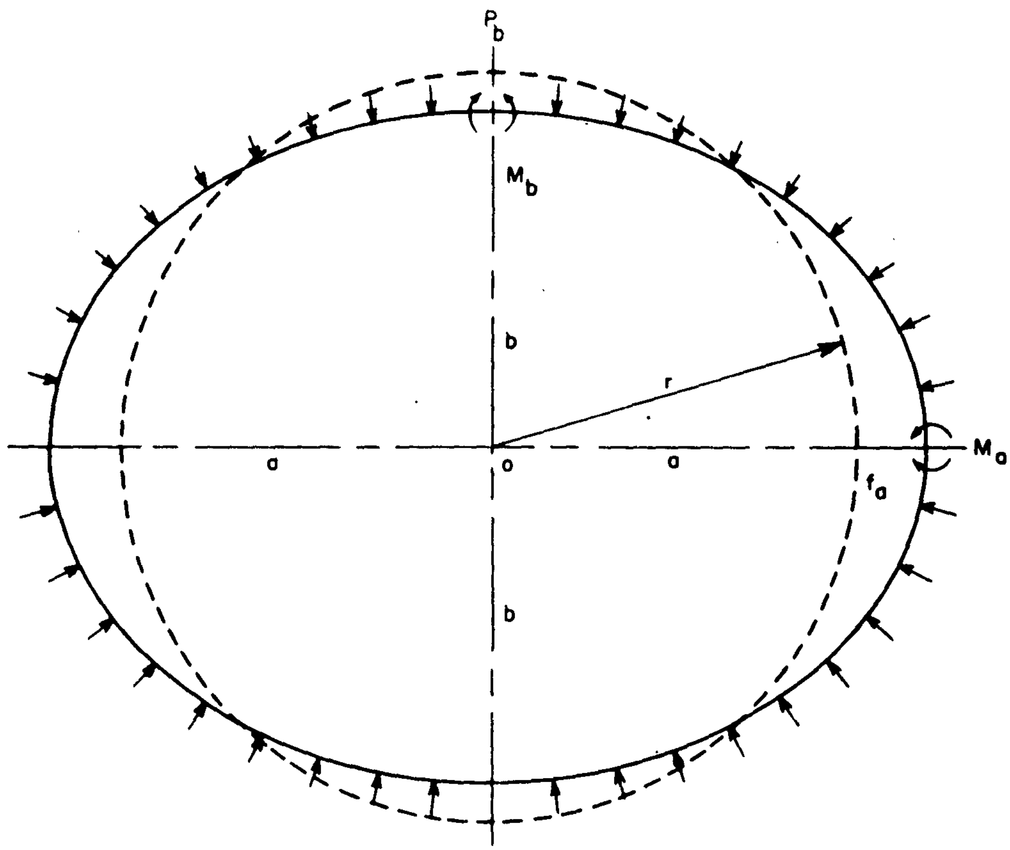


Figure 10

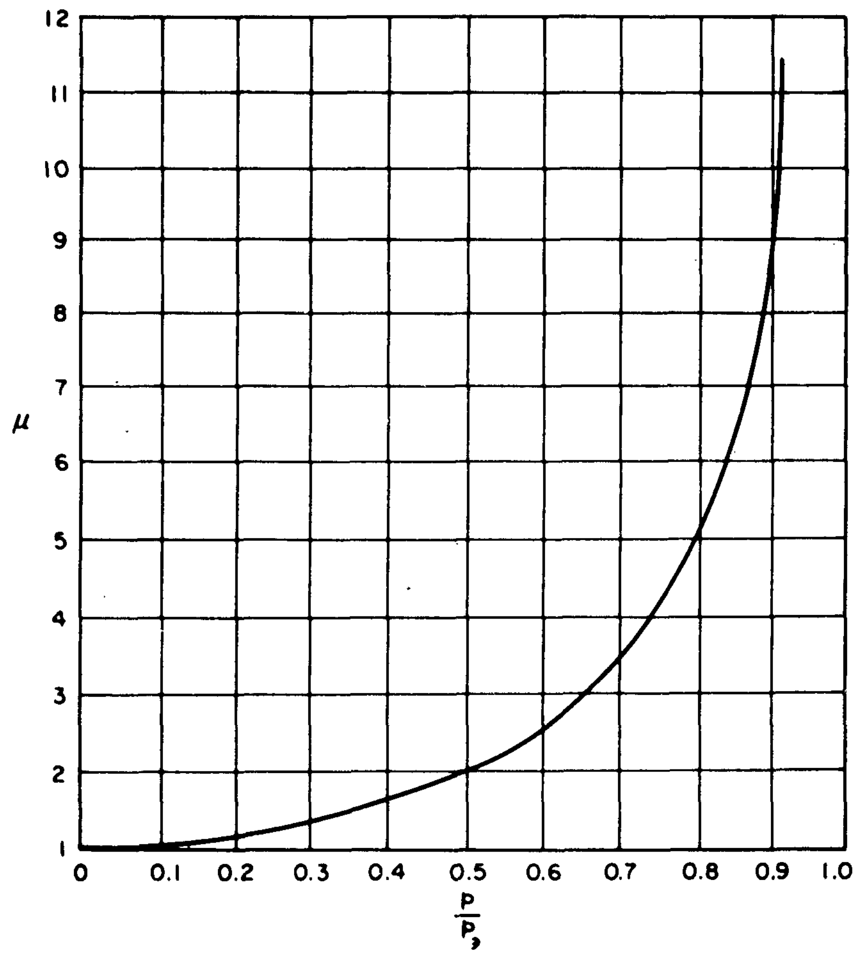
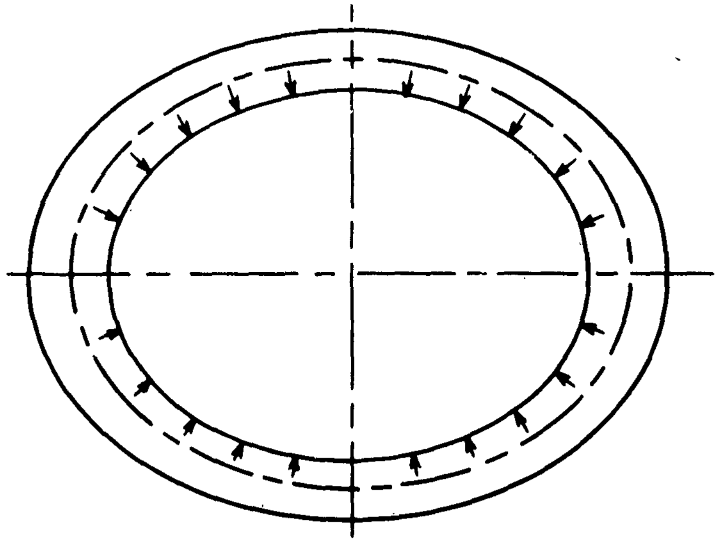


Figure 11

Figure 12



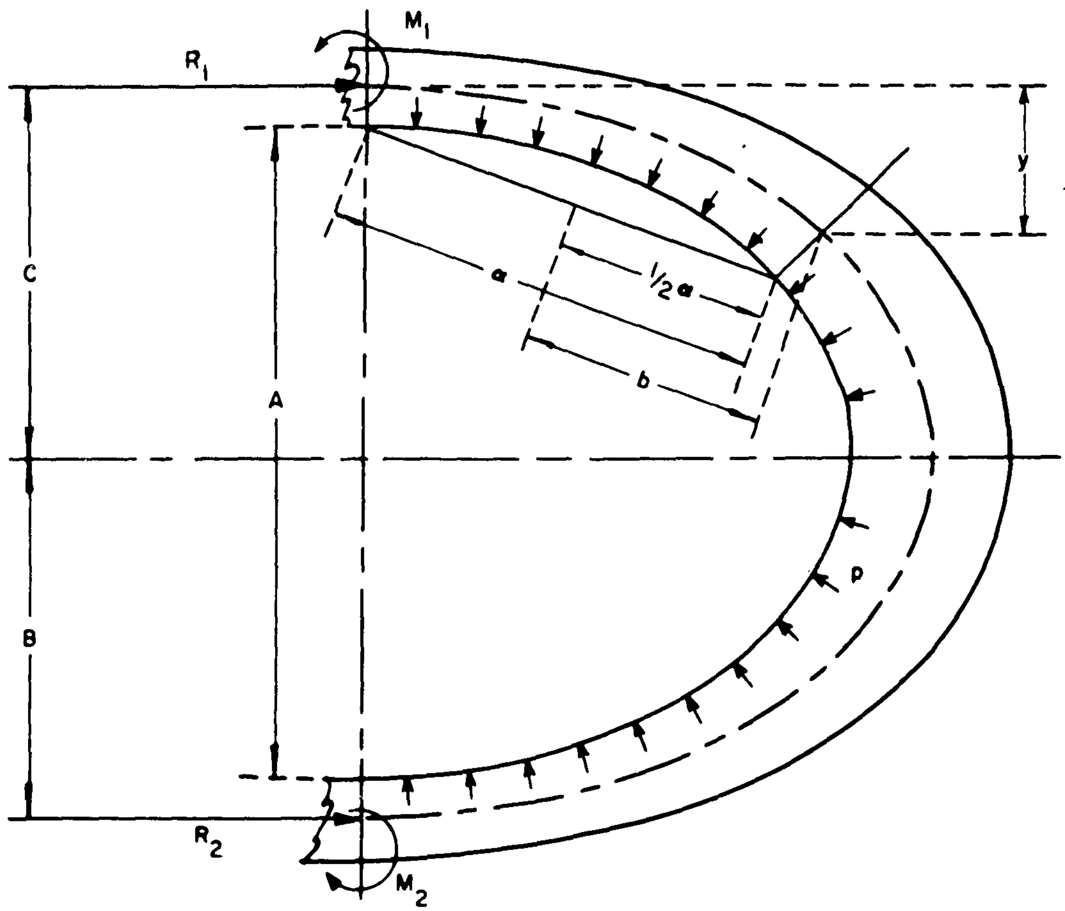


Figure 13

TABLE 12

Name of Object

Radius of Hull: $r = 200$ cm	Thickness of Plating $t = 1.25$ cm	Length of Space [between frames] $l = 70$ cm	
Profile of Frame: Channel Iron No. 16	Sectional Area: $F = 24.9\ \text{cm}^2$	Position of Neutral Axis $e = 8.0$ cm	Natural Moment of Inertia $i = 954$ cm

		I	II	III	IV	V
Calculation of Moment of Inertia for Frame:	Composition of Section	Shoulder or Arm*	Area	Static Moment Column I × II	Transferable Moment of Inertia Column $I^2 \times II$	Natural Moment of Inertia
Sketch	Strake of Plating 1.25 × 70	0	87.5	0	0	0
	Channel Iron 16	8.6	24.9	214	1844	954
	Σ	1.9	112.4	214	1844**	954

Moment of Inertia: $I = 1844 + 954 - \frac{214^2}{112.4} = 2390$ cm
Critical Pressure: $\frac{2EI}{r^3 l} = 17\ \text{atm} > q_p$
Stress in Frame: $E\frac{w_0}{r}$ = Line [13] of Table 1 = $1070 < 0.35\,\sigma_T$

EDITOR's NOTES:

*Probably a certain distance. In the text the author says it is "a portion of the plating equal to the length of the space," but this was given above as 70 cm.

**Should be about 407 unless this is the sum of above two items in which case the value for shoulder Σ = 1.9 is wrong.

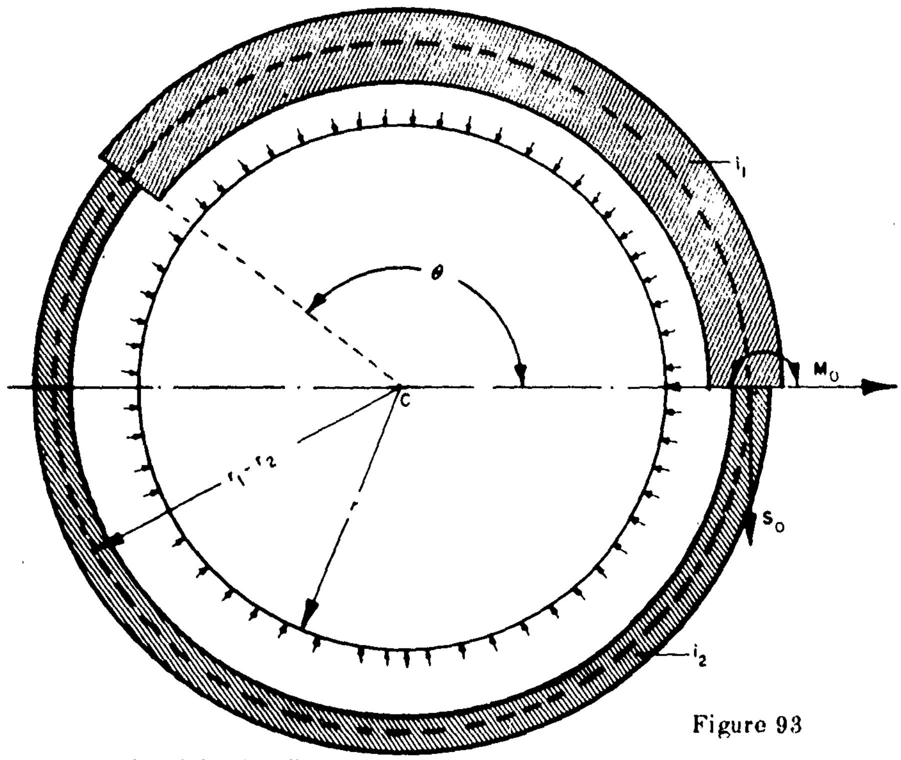


Figure 93

Note: Figures 93 thru 96 (from Part 2) are placed here for the reader's convenience.

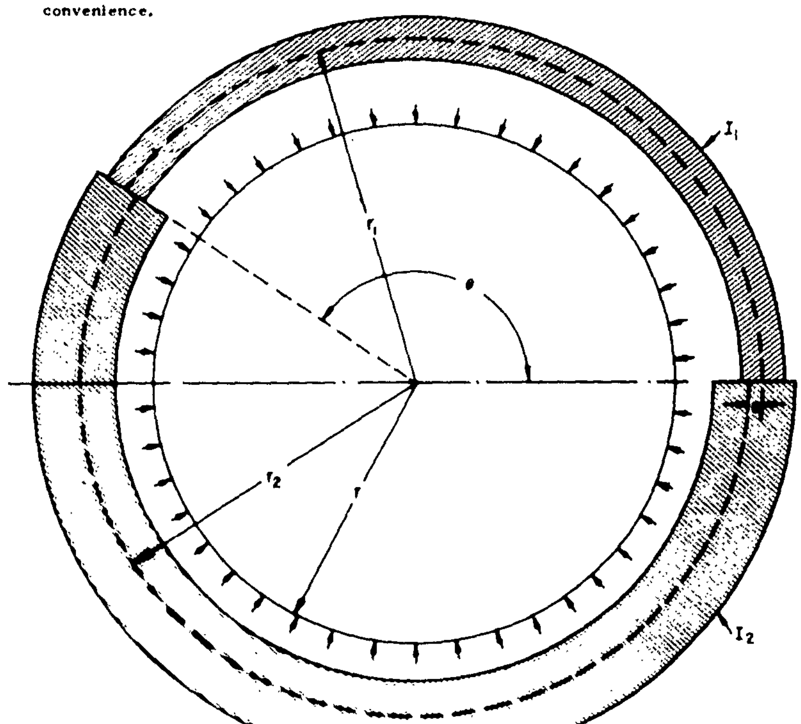


Figure 94

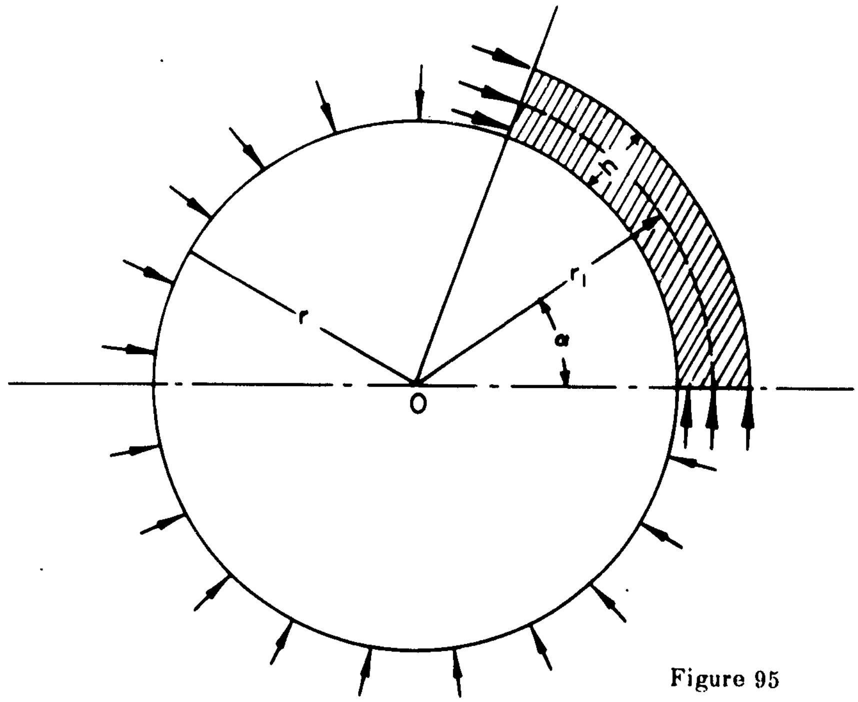


Figure 95

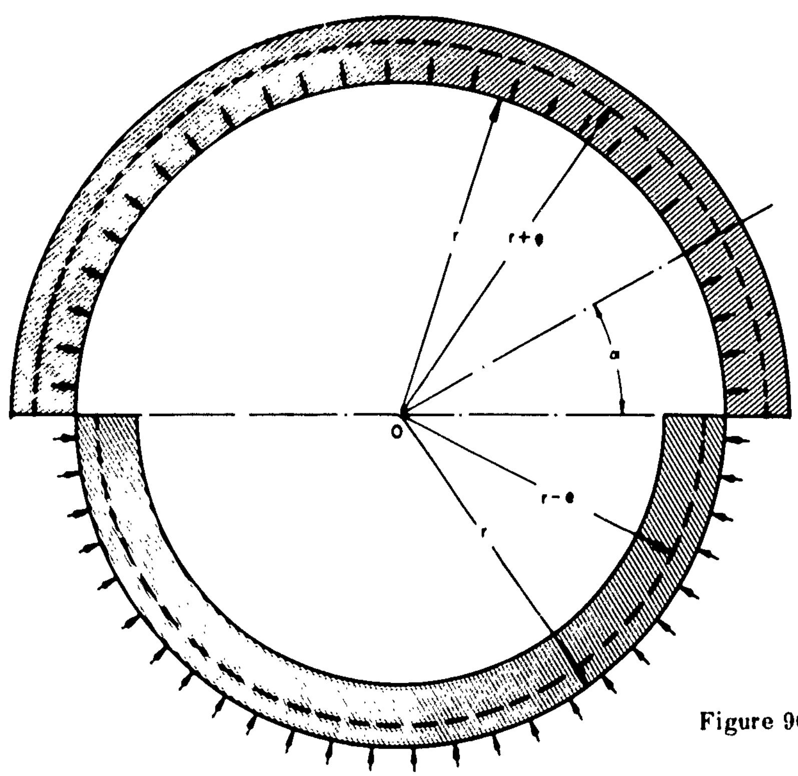


Figure 96

CHAPTER 4

STRENGTH ANALYSIS OF COMPOSITE FRAMES*

The composite frames of the pressure hull of a submarine are usually formed by separate parts of a circular or other shape, joined to each other at the points of intersection by clamps or cleats; in the spaces between the joints, there are supplementary connections of one type or another. The external, uniformly distributed load acting on the frame is applied either along the outer or along the inner contours of the separate parts of the frame.

Frames of such design are usually mounted in the vicinity of the various tanks of the pressure hull (Figures 14 to 17), but under certain conditions, it may be found expedient to employ composite frames even for the structure of the entire pressure hull (Figures 18 and 19).

Before passing to the exposition of the strength analysis of composite frames, we must first enumerate certain propositions, or lemmas, used in these calculations.

13. PRELIMINARY PROPOSITIONS.

1. If there is a segment of a straight line AB, of length s and a force equal to P = ps, directed normally with respect to AB, then the components of the force P in any two given directions XX and YY are consequently equal to (Figure 20):

$$X = py \qquad Y = px$$

where x and y are equal to the components segment AB for directions normal to those given. This proposition results directly from the similitude of figures obtained by a geometric resolution of the force P and the segment AB in the given directions.

In particular, if the given directions are mutually perpendicular, the quantities x and y appear as projections of AB in these directions (Figure 21).

2. In order that a portion of the arc of radius r, subject to a uniformly distributed load p, be in a state of equilibrium and not undergo bending, it is necessary and sufficient that reactions, equal to R = pr and directed along tangents to the arc, be applied at the ends of the arc

*Shimansky, Yu. A.: "Calculation of Composite Frames for the Hull of a Submarine", Collection HNBK (NIVK), Number 6, 1938.

(Figure 22).

Resolving these reactions in the direction of the chord and normal to it, as shown in Figure 22, we get:

component in the direction of the chord:

$$X = pr \cos \alpha = py$$

component in the direction normal to the chord:

$$Y = pr \sin \alpha = px$$

where $y = r \cos \alpha$ and $x = r \cos \alpha$ are equal to the projections of the radius passing through the ends of the arc on directions of the corresponding diameter or, what amounts to the same thing, equal to the coordinates of the end of the chord.

Thus, the magnitude of the general reaction pr of the arc in the direction of the tangent does not depend on the length of the arc, i.e., on the magnitude of the central angle corresponding to it. However, the magnitudes of the components of the reaction (py and px) in the direction of the chord and normal to it, do depend on the length of the arc, or, in other words, on the magnitude of the central angle.

From what has been stated, it follows that any part of the arc may be substituted by a force, equal to pr and directed along the tangent at the point of separation, without change of the forces found previously for the ends of the arc: pr, or py and px (Figure 23).

However, it must be taken into account that the ends of the arc which are a part of the circle of radius r, loaded by uniform load p, must, as the result of the application in its sections to axial forces pr, undergo displacements in the direction of the radii equal to:*

$$\Delta_0 = \frac{pr}{EF} r$$

and consequently, the displacements in the direction of the chord are equal to $\delta_0 = \Delta_0 \sin \alpha$, where F denotes the area of the section of the arc (Figure 24). Only when such deflections occur, is it practically possible to consider that the arc will not undergo bending, but only be subject to axial forces equal to pr. If, in reality, as a result of some structural conditions, the displacement of the end of the arc in the direction of the chord proves to be equal to δ_1, and, as a consequence the

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*Editor's Note: The following equation was badly typeset in the original and could not be read. We have indicated by r those symbols we could not read.

displacement in the direction of the radius is equal to:

$$\Delta_1 = \frac{\delta_1}{\sin\alpha}$$

then the regular circular contour of the arc will be distorted; the greatest deflection resulting from the bending will then be $\Delta_1 - \Delta_0$. In sections of the arc, bending moments will occur; the largest of these bending moments at the end of the arc will be equal to:

$$\mathfrak{M} = pr(\Delta_1 - \Delta_0) = pr\left[\frac{\delta_1}{\sin\alpha} - \frac{pr^2}{EF}\right].$$

The greatest value of this moment, assuming absolute rigidity of the cross-member ($\delta_1 = 0$), is:

$$\mathfrak{M} = -\frac{p^2r^3}{EF}$$

3. If, on a beam AB of arbitrary shape, whose length is s, a uniformly distributed load p is applied, the equivalent force of this load, equal to ps, passes through the center of the beam in a direction normal to it (Figure 25).

The components of this load in the two mutually perpendicular directions are:

$$\overline{X} = py \quad \text{and} \quad \overline{Y} = px$$

where x and y are the projections of the beam in the given directions.

A uniformly distributed load p, acting on a beam of any shape AB, is equivalent to the uniformly distributed load of the same intensity, acting on an arbitrary system of intersecting beams BCDA passing through the ends of the beam being examined (see Figure 26). Therefore, from the point of view of statics, a uniformly distributed load, acting on a beam, can be resolved into components of a load of the same intensity acting in any two or more directions passing through the ends of the beam under investigation.

4. If, in a circular beam subjected to a uniform load p, a certain portion is removed and replaced by a rigid cross-member, the forces and moments in the section of the remaining circular beam will not change, whereby at the points of juncture to the cross-member, shear forces will be acting equal pr (Figure 27). The components of these forces in the direction of the cross-member and normal to it will be (see Paragraph 1 of the present section):

$$\overline{X} = py \quad \text{and} \quad \overline{Y} = px$$

where x and y are coordinates of the joints

In this case, the cross-member will behave as a freely supported

beam supported at two points under the application of a uniform load p and axial forces equal to $\overline{Y} = px$, where x is equal to the distance from the center of the circumference of the beam O to the cross-member.

5. If the structure being examined is composed of two intersecting circular beams and a rigid cross-member situated between the points of their intersection (joints), the forces acting on the cross-bar are reduced to only axial forces, applied along the line of juncture and equal to (Figures 28 and 29):

$$\overline{Y} = p(x + x_1) = p \times \overline{OO}_1$$

where $\overline{OO}_1$ is the distance between the centers of the circular beams.

The components of the forces in the direction normal to the line of juncture, $\overline{X} = py$, mutually equilibrate each other.

Thereby, it is evident that as the center O_1 passes to a point beyond the center O on the side opposite the location of the cross-member, the axial force in the cross-member will change its sign.

6. The arc of a segment of a circle, or a full circle loaded by a uniform pressure p, may be replaced by any jointed or hinged polygon without disturbing the equilibrium of the system, whereby along the sides of this polygon there will be acting, in addition to the pressure p, axial forces S equal to the product of the intensity of the pressure p and the distance from the corresponding side of the polygon to the center of the circle (Figure 30); for example,

$$S = p \times \overline{O}\, a$$

The arc of a portion of a circle, or of a full circle, loaded by a uniform pressure p may, without disturbing the equilibrium, be replaced by a system of arcs of arbitrary radii inscribed within the given circle or arc.

In this case, both in the arcs and in the cross-members, only axial forces will be developed and will be given, respectively, by

$$S_1 = pr \quad \text{and} \quad S_2 = p \times \overline{OO}_1$$

wherein r is the length of the radius of the arc and $\overline{OO}_1$ is the distance from the center of the arc to the center of the circle (Figure 31).

A uniformly distributed load acting on the above described hinged systems, consisting of rigid junctures, insofar as axial forces are concerned, may be applied uniformly on them only if all the joints of the hinged system be on one circle. Since a circle may be described about any triangle, the hinged system under investigation, having the form of any triangle, can consequently serve to equilibrate a uniformly distributed load (Figure 19).

7. If the beam ABC (Figures 32 and 33), having a partially circular shape of radius r, is loaded by a uniformly distributed load p and an axial

force pr, applied to the circular end of beam A, the indicated external forces may be reduced for any section of beam C to one concentrated force normal to the radius through the point C and equal to P = ps, where s is the distance from point C to the center; the point of intersection E of the direction of this force with the radius has a distance from point C equal to

$$e = \frac{1}{2}s(r^2 - s^2)$$

In fact, on the basis of Paragraph 3 of the present section, a load acting on part of the beam BC may be replaced by loads on the segments BD and DC where BD represents the prolongation of the arc and DC is the direction of the radius drawn toward point C. The load on the portion BD, together with the load acting on the circular part of beam AB, may be replaced by the concentrated force pr, applied at point D, normal to the radius (see Paragraph 2 of this section); the load on the segment DC equal to k = r - s may be replaced by the concentrated force pk normal to the radius and applied at the center of the portion DC.

Adding the forces pr and p(r - s) according to the rule of summation of parallel forces, we shall get a resultant force P = ps applied at point E, whereupon we find that

$$EC = e = \frac{1}{2}s(r^2 - s^2)$$

The force so found P = ps, equivalent to the load acting on the beam ABC, may thereupon be reduced to any point and decomposed in any direction. Thus, taking point C as the base point of reduction, we shall get for the bending moment;

$$\mathfrak{M} = Pe = ps \times \frac{1}{2s}(r^2 - s^2) = \frac{p}{2}(r^2 - s^2)$$

and the concentrated force P = ps normal to the radius through the point C; for the resolution of this concentrated force in any two chosen directions, we have to resolve the segment OC = s into directions normal to those chosen, as outlined in Paragraph 1 of this section.

14. ANALYSIS OF FRAMES CONSISTING OF INTERSECTING CIRCULAR BEAMS AND RECTILINEAR CROSS BRACES (Figures 18 and 19)

As a peculiarity of this type of frame the clear distribution of the functions performed by their composite members, may be mentioned i.e. by the circular beams and straight cross braces, and, as a consequence the absence of bending stresses in sections of these members. The circular beams take up the external load acting on the frame and transmit it as concentrated shear forces to the joints; the cross-members, located between these joints, act to equilibrate the concentrated forces mentioned.

The axial force in a circular beam is:

$$S = pr$$

The axial force in the cross-member (Paragraph 5 of the preceding section) is:

$$S = p \times \overline{OO}_1$$

where p is the intensity of the external load; r is the radius of the circle of the circular beam under examination; $\overline{OO}_1$ is the distance from the center of the circumference of that circular beam, which is closed by the cross-member under investigation, to the center of the circle on which the joints of the frame are situated.

The analysis of circular beams and cross-braces subjected to the axial forces produced in them must be performed both with respect to strength and stability, proceeding from the condition of equal strength. For this purpose the usual formulas are used, considering the ends of the beams and cross-braces as freely supported.

15. ANALYSIS OF FRAMES COMPOSED OF CIRCULAR BEAMS AND CROSS BRACES OF ARBITRARY SHAPE UNDER EXTERNAL LOADING.

To frames of the type mentioned in the heading belong all varieties of composite frames encountered in practice, wherein the transverse braces serve as connections which not only equilibrate the external loading but partially absorb it (Figures 14, 15, 16, 17).

A brace may be considered (as outlined in Section 13, Paragraph 4), for purposes of analysis, equivalent to a simply supported beam resting on two supports subjected, in addition to the distributed load p, to a compressive force acting along a line connecting the supporting sections, equal to px, where x is the distance from this line to the center (Figures 34 and 35, left half).

The order of further analysis of the beam is determined by its design; the following three structural variants of such beams usually encountered in practice should be mentioned.

1. The beam is in the shape of a truss girder. The forces acting in the members of the beam may be found with the aid of the usual methods used in truss analysis.

In determining the stresses in the members, their deflection must be considered as well as the distributed load in the span.

2. The beam is a solid web or a web with small cut outs as in Figures 14 and 17, right side. The forces and moments in the section of such a beam are calculated most simply according to the procedure outlined in Section 13, Paragraph 7.

For example, let it be required to calculate the forces and moments for the section of the beam passing through point C of the loaded edge (Figures 34, 35, 36). For this purpose, we draw a radius through point C and mark on it a point E, at a distance from C equal to

$$e = \tfrac{1}{2} s (r^2 - s^2)$$

where s is the length of the portion OC.

The forces and moments in the section being examined are reduced to a single force equal to P = ps, passing through the point E normal to the radius.

The components of this force, reduced at the center of gravity of the section, will be;

bending moment; $\mathfrak{M}$ = p s i,

where i is the distance from the direction of the force P to the center of gravity; axial force S = px and shearing force N = py, where x and y are the components of the segment s in directions along and normal to the section, (Section 13, Paragraph 1).

In Figure 35, the component of force P in the direction of the section equals zero; in the case of the type of braced beam shown on the figure, the axial force and the bending moment for all sections of the beam are the same.

Having found the bending moment $\mathfrak{M}$, the axial force S and the shearing force N, acting in the section of the beam, it is possible, by the usual formulas, to calculate the maximum stresses in the section of the beam under investigation.

3. The beam is designed as a truss girder without cross-stays (see Figures 15, 16, and the left halves of Figures 14 and 17). Every portion of such a beam represents a closed frame ABCD, consisting of four members with rigid joints, whereby the two opposite bars of this frame BC and AD are absolutely rigid whereas the two remaining members AB and CD have terminal rigidity only (see Figures 37, 38, 39, 40).

The external load acting on this frame consists of the distributed load p applied on one of the members having terminal rigidity and the forces acting on the absolutely rigid members. The latter are equivalent to the forces and moments in the sections of the beam which limit the segment of it under investigation. Every one of these forces, as was shown previously, is reduced to one concentrated force equal to P = ps, normal to the radius drawn through joint C of the loaded member and, at a distance from this joint equal to

$$e = \tfrac{1}{2} s (r^2 - s^2)$$

where s is the distance between this joint and the center of the circular frame of radius r.

For the analysis of such a frame, we shall consider its lower rigid member as immovable and the force $P = ps$, applied to the upper rigid member, will be reduced to point C, resolving it in the direction of the member CD and normal to the latter.*

As a result of such a reduction, we shall get at point C:
bending moment:

$$\mathfrak{M} = Pe = \frac{1}{2} p(r^2 - s^2) \qquad [1]$$

the force in the direction of the member CD:

$$S_0 = p \times \overline{OF} \qquad [2]$$

the force in the direction normal to CD:

$$N = p \times \overline{CF} \qquad [3]$$

where $\overline{OF}$ and $\overline{CF}$ are equal to the components of the segment $\overline{OC} = s$ in the direction and normal to CD, respectively.

Let us take for positive directions of the external forces and moments $\mathfrak{M}$, N, S_0 and p acting on the frame, and likewise, for positive directions of the initial deflection of the members, their directions shown in Figure 39.

Let us introduce the following notation: l_1 and l_2 = the length of members AB and CD; f_1 and f_2 = the initial deflections of the members, considering them as positive for the type of bending shown in Figure 39. The magnitudes of the deflections can be measured either according to the sketch or calculated according to the well-known expression $f = l^2/8R$ where l is the length of the member and R is the radius of curvature; α is the angle between the members AB and CD whereupon

$$\sin \alpha = h_1/l_1\,;\ \cos \alpha = h_2/l_1 \text{ where } h_1 = \overline{AK} \text{ and } h_2 = \overline{BK}$$

In this expression, point K is the intersection of lines drawn from the joints A and B normal and parallel to the directions of member CD; a and b are the lengths of the portions CL and BL where L is the point of intersection of the direction of member AB with the perpendicular to this direction

*If the neutral axis of the member CD does not noticeably coincide with the line of application of the external load p, the force $P = ps$ must be brought to a point lying on the neutral axis of this member.

dropped from point C. I_1, I_2 are moments of inertia of the sections of members AB and CD;

$$\left.\begin{array}{l}
\mu_1 = \frac{2}{3} f_1 \qquad \mu_2 = \frac{2}{3} f_2 \qquad t = \frac{h}{a} \qquad \gamma = \frac{l_2}{l_1} \\[2ex]
\eta = \frac{h}{a} \sin \alpha + \cos \alpha \qquad k = t\gamma \qquad \chi = \gamma^2 \frac{I_1}{I_2} \\[2ex]
A = \frac{2(1.5 + k)}{3(2 + k)} l_2 \qquad B = \frac{\frac{3}{2} + k}{4(2+k)} l_2^2 \\[2ex]
C = (2t + 3\eta)\chi \qquad D = \frac{3(t + 2\eta)}{(2t + 3\eta) l_1} \\[2ex]
T = \mathfrak{M} + AN + \mu_2 S_0 \mp B p \\[2ex]
K = a + A \sin \alpha + \mu_2 \cos \alpha + \mu_1 \\[2ex]
L = b - A \cos \alpha + \mu_2 \sin \alpha + \frac{1}{D}
\end{array}\right\} \text{conventional notation}$$

m_1, n_1, S_1 are bending moment, shearing force and axial force respectively acting in the upper section of member AB;
m_2, n_2, S_2 are bending moment, shearing force and axial force respectively acting in the upper section of member CD.

For positive directions of forces and moments, we shall take their directions as shown in Figure 41.

If the forces and moments acting in the upper sections of members AB and CD are known, the forces and moments in the middle and lower sections of these members can be calculated according to the following expression, resulting directly from examination of Figure 41 (the axial forces remain constant).

For member AB:

in the middle section:

$$m_c = m_1 + \frac{1}{2} n_1 l_1 - S_1 f_1 \qquad n_c = n_1 \tag{4}$$

in the lower section:

$$m_3 = m_1 + n_1 l_1 \qquad n_3 = n_1 \tag{5}$$

For member CD:
in the middle section:

$$m_c' = m_2 - \frac{1}{2} n_2 l_2 + S_2 f_2 \pm \frac{1}{8} p l_2^2$$

$$n_c' \geq n_2 \mp \frac{1}{2} p l_2 \qquad [6]$$

in the lower section:

$$m_4 = m_2 - n_2 l_2 \pm \frac{1}{2} p l_2^2$$

$$n_4 = n_2 \mp p l_2 \qquad [7]$$

Calculation of a frame without cross-stays (diagonals) must be made on the basis of the following expressions, employed for determination of the unknown forces and moments in the upper sections of members AB and CD, and in the following order:

a) According to the diagram of the frame, the lengths of the segments $\overline{OF}$, $\overline{CF}$ and $s = \overline{OC}$ are found, which determine the magnitudes of the external moments and forces, $\mathfrak{M}$, N and S_o (according to Equations [1], [2] and [3]) and similarly, the other dimensions of the frame necessary for calculation as mentioned in the notation.

b) The numerical values are calculated according to the formulas given above, taking into account the directions (signs) of the deflections and of the external loads acting on the frame.

c) The unknown forces and moments acting in the upper sections of members AB and CD are calculated in first approximation, assuming $n_1 = 0$, according to the following expressions:

$$\left.\begin{aligned}
n_1 &= 0 \\
S_1 &= T/K \\
S_2 &= S_1 \cos \alpha - S_0 \\
n_2 &= N - S_1 \sin \alpha \\
m_2 &= A n_2 - S_2 \mu_2 \mp B p \\
m_1 &= S_1 \mu_1 + C\left[m_2 - \frac{2}{3} l_2 n_2 + S_2 \mu_2 \pm \frac{1}{4} p l_2^2\right]
\end{aligned}\right\} \qquad [8]$$

The values for the forces and moments obtained in first approximation are written in the first column of Table 13.

d) The correction to the first approximation of the force n_1 is calculated according to

$$\Delta_1 n_1 = D(S_1 \mu_1 - m_1) \qquad [9]$$

and the correction for the other forces and moments is calculated according to the expressions:

$$\left.\begin{aligned}
&\Delta_1 S_1 = \frac{L}{K}\Delta_1 n_1 \\
&\Delta_1 S_2 = \Delta_1 S_1 \cos\alpha - \Delta_1 n_1 \sin\alpha \\
&\Delta_1 n_2 = -\Delta_1 S_1 \sin\alpha - \Delta_1 n_1 \cos\alpha \\
&\Delta_1 m_2 = A\,\Delta_1 n_2 - \Delta_1 S_2 \mu_2 \\
&\Delta_1 m_1 = \mu_1 \Delta_1 S_1 + C\left[\Delta_1 m_2 - \frac{2}{3} l_2 \Delta_1 n_2 + \mu_2 \Delta_1 S_2\right]
\end{aligned}\right\} \qquad [10]$$

The numerical values found for the corrections are written in the second column of Table 13.

e) The second correction for the force n_1 is calculated according to:

$$\Delta_2 n_1 = D(\Delta_1 S_1 \mu_1 - \Delta_1 m_1) \qquad [11]$$

and likewise the coefficient for total correction required for the following calculation is given by

$$\mu = \frac{\Delta_1 n_1}{\Delta_1 n_1 - \Delta_2 n_1} \qquad [12]$$

The values for the corrections listed in the second column of Table 13 are multiplied by the coefficient of total correction μ; these results are tabulated in the third column of the table.

The derived values for the forces and moments are found to be the result of addition of the values given in the first and third columns of the table; these results are written in the fourth column of the table.

f) The forces and moments applied in the middle and lower sections of members AB and CD are calculated according to Equations [4] to [7] and the strength of these members is checked with respect to stresses.

In carrying out the calculations, the following must be kept in

mind.

(i) Since, in the equations given above, the external load p acting along the circumference of the frame enters into the calculation as a multiplier, it can be dropped from all intermediate computations and be reserved merely to multiply the final result of calculation. If, in doing so, the load p acts along the outer edge of the loaded member CD, numbers having the multiplier p in the expressions under Equation [8] must be taken with the lower signs* (Figure 40).

(ii) For a check of the correctness of the arithmetical computations the expression may be used:

$$S_1 a - \mathfrak{M} - m_2 + m_1 - n_1 b = 0 \qquad [13]$$

in which the values of the forces and moments, given in the final column of Table 13, must be substituted.

(iii) In checking the strength of the sections of members AB and CD, according to the forces and moments determined in them, the following standards for allowable stresses must be taken, under the condition that the loading p corresponds to the design pressure.

The standard for the general stresses (in middle sections of the members): -- up to the yield point of the material.

The standard for local stresses (in the end sections of the members): for tensile stresses -- 125% of the yield limit of the material; for compressive stresses -- 100% of the yield limit of the material.

(iiii) For determination of the total stresses in sections of the members, the directions (signs) of the bending moments and axial force must be considered (in Figure 41, the directions of these forces and moments are shown as positive).

4. The optimum design of a girder without cross-stays or diagonals. The optimum type of girder, without cross-stays, investigated in the preceding section, must meet the condition consisting of the absence in its members of the end bending moments m and of shearing forces n.

We shall show that, to satisfy this condition, it is necessary and sufficient that all four joints of each frame of the girder lie upon the same circle (Figure 42).

Let us examine one of the frames, for example ABCD, of such a girder without cross-stays, shown in Figure 37. Let us apply to the edge of the girder without loading equal and oppositely directed, distributed

*Editor's Note: Apparently, as the values concerned in Equation [8] are provided with the ± sign, the author means that the negative sign applies.

loads +p and -p.

The load +p, acting over the entire circumference of the frame, produces a compressive force pr in the member AB.

The load -p, acting on parts BN and AM, and the load +p which acts on the segment CN and DM (Figure 37), may be replaced by loads +p acting along the sides BC and AD.

Thus, the load acting on the frame of a girder being investigated, is equivalent to the uniformly distributed loads +p, applied to all four sides of the frame. If all the joints of the frame are on one circle, these loads are equivalent to a uniformly distributed pressure of the same intensity acting along the circumference of the circle (Figure 42). In relation to this circumference, the members of the frame appear as rigid cross-braces, loaded only by axial forces equal to the product of the pressure p and the distance to the cross-brace from the center of the circle (Section 13, Paragraph 6).

From all that has been said, it is evident that the frame under investigation, which represents a hinged quadrangle inscribed within the circle, will not be distorted. In member AB, there will act a compressive force pr, as well as a compressive force and bending moment due to the axial force equal to ps, where s is the distance to the member from the center of the circle. In member CD there will act a compressive force and a bending moment due to the axial force ps_1 where s_1 is the distance of this member from the center of the circle and bending moments due to the application of a uniformly distributed load p.

EXAMPLES*

Example 1.

To find the forces and moments in sections of a member of a frame without cross-stays (Figure 43), which represents a part of a composite frame such as shown in Figure 14, according to the following data.

radius of the circumference of the frame $r = 2.25$ [m]
radius of the inner member $R = 3.25$ [m]

$$l_1 = AB = 1.62[m] \quad l_2 = CD = 1.55[m] \quad \lambda_1 = AK = 0.98[m]$$

$$\lambda_2 = BK = 1.39[m] \quad a = CL = 1.17[m] \quad b = BL = 0.45[m]$$

the initial deflection of the unloaded member AB

$$f_1 = \frac{l_1^2}{8r} = \frac{(1.62)^2}{8 \times 2.25}$$

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*Editor's Note: Inasmuch as t and m are both used as symbols, the abbreviations for meter [m] and ton [t] will be placed in brackets.

similarly, for the loaded member:

$$f_2 = \frac{l_2^2}{8R} = \frac{(1.55)^2}{8 \times 3.25} = 0.0925 \text{ [m]}$$

The direction of both deflections is positive. The intensity of the load on the frame is p in [t] per running meter (the load on the member CD is positive in direction).

The moment of inertia of sections AB and CD are identical ($I_1 = I_2$).

The external loading acting on the frame is equivalent to one concentrated force P = ps, normal to the radius drawn from the center O through C and distant from C by the quantity $e = \overline{CE} = 1/2s\,(r^2 - s^2)$ where s is the length of the segment $\overline{OC}$, i.e. s = 1.56 [m].

Reducing this force to point C and resolving it in the direction of and perpendicular to member CD, we get (instead of decomposition of the force, we decompose the segment CD):

bending moment according to Equation [1]:

$$\mathfrak{M} = Pe = \frac{1}{2}p(r^2 - s^2) = \frac{1}{2}p(2.25^2 - 1.56^2) = 1.315\,p \;[\text{t} - \text{m}]$$

force in the direction of member CD according to Equation [2]:

$$S_0 = p \times \overline{OF} = 0.04\,p \;[\text{t}]$$

force in the direction normal to member CD by Equation [2]:

$$N = p \times \overline{CF} = 1.58\,[\text{t}]$$

The direction of the moments and forces $\mathfrak{M}$, S_0, N, is positive.

Computing the numerical values of the quantities:

$$\sin\alpha = \frac{h_1}{l_1} = \frac{0.98}{1.62} = 0.605 \qquad \cos\alpha = \frac{h_2}{l_1} = \frac{1.32}{1.62} = 0.815$$

$$\mu_1 = \frac{2}{3}f_1 = \frac{2}{3} \times 0.146 = 0.0975 \qquad \mu_2 = \frac{2}{3}f_2 = \frac{2}{3} \times 0.0925 = 0.0615$$

$$\iota = \frac{h_1}{a} = \frac{0.98}{1.17} = 0.837 \qquad \gamma = \frac{l_2}{l_1} = \frac{1.55}{1.62} = 0.957$$

$$k = \iota\gamma = 0.802 \qquad \chi = \gamma^2\frac{I_1}{I_2} = 0.915$$

$$\eta = \frac{b}{a}\sin\alpha + \cos\alpha = 1.047$$

$$A = \frac{2(1.5+k)}{3(2+k)} l_2 = 0.85 \qquad B = \frac{1.333+k}{4(2+k)} l_2^2 = 0.457$$

$$C = (2t + 3\eta) X = 4.41 \qquad D = \frac{3(t+2\eta)}{(2t+3\eta) l_1} = 1.12$$

$$T = \mathfrak{M} + AN + \mu_2 S_0 - B = 2.203$$

$$K = a + A \sin\alpha + \mu_2 \cos\alpha + \mu_1 = 1.832$$

$$L = b - A\cos\alpha + \mu_2 \sin\alpha + \frac{1}{D} = 0.688$$

We compute the value of the stresses in first approximation according to Equation [8] as:

$$n_1 = 0$$

$$S_1 = \frac{T}{K} = \frac{2.203}{1.832} = 1.202$$

$$S_2 = S_1 \cos\alpha - S_0 = 1.202 \times 0.815 - 0.04 = 0.94$$

$$n_2 = N - S_1 \sin\alpha = 1.58 - 1.202 \times 0.605 = 0.852$$

$$m_2 = An_2 - S_2\mu_2 - B = 0.850 \times 0.852 - 0.94 \times 0.0615 - 0.457 = 0.210$$

$$m_1 = S_1\mu_1 + C\left[m_2 - \frac{2}{3} l_2 n_2 + S_2\mu_2 + \frac{1}{4} l_2^2\right] = 0.059$$

The values obtained will be entered in the first column of Table 14.

We compute the corrections to the first approximation by Equations [9] and [10] as follows:

$$\Delta_1 n_1 = D(S_1\mu_1 - m_1) = 1.12\,(1.202 \times 0.0975 - 0.059) = 0.0652$$

$$\Delta_1 S_1 = \frac{L}{K}\Delta_1 n_1 = \frac{0.688}{1.832} \times 0.0652 = 0.0245$$

$$\Delta_1 S_2 = \Delta_1 S_1 \cos\alpha - \Delta_1 n_1 \sin\alpha = 0.0245 \times 0.815 - 0.0652 \times 0.605 = -0.0195$$

$$\Delta_1 n_2 = -\Delta_1 S_1 \sin\alpha - \Delta_1 n_1 \cos\alpha = 0.0245 \times 0.605 - 0.0652 \times 0.815 = -0.068$$

$$\Delta_1 m_2 = A\Delta_1 n_2 - \Delta_1 S_1 \mu_2 = -0.85 \times 0.068 + 0.0195 \times 0.0615 = -0.0566$$

$$\Delta_1 m_1 = \mu_1 \Delta_1 S_1 + C\left[\Delta_1 m_2 - \frac{2}{3} l_2 \Delta_1 n_2 + \mu_2 \Delta_1 S_2\right] = 0.975 \times 0.0245$$

$$+ 4.41\left[-0.0566 + \frac{2}{3} 1.55 \times 0.068 - 0.0615 \times 0.0195\right] = 0.0571$$

We enter the values obtained in the second column of Table 14.

Next, we compute the second correction $\Delta_2 n_2$ by Equation [11] and the coefficient of total correction by Equation [12]:

$$\Delta_2 n_1 = D(\Delta_1 S_1 \mu_1 - \Delta_1 m_1) = 1.12\,(0.0245 \times 0.0975 - 0.0571) = -0.0613$$

$$\mu = \frac{\Delta_1 n_1}{\Delta_1 n_1 - \Delta_2 n_1} = \frac{0.0652}{0.0652 + 0.0613} = 0.515$$

Multiplying the values in the second column of Table 14, by $\mu =$ 0.515, we enter the results in the third column of the table. Then, adding the figures of the first and third columns, we enter the result in the fourth column.

The degree of accuracy found for the values of forces and moments is checked by Equation [13]:

$$S_1 a - \mathfrak{M} - m_2 + m_1 - n_1 b = 1.215 \times 1.17 - 1.315 - 0.181 + 0.0884 - 0.0336 \times 0.45 = 1.5104 - 1.5111$$

Having found the forces and moments in the upper section of the members AB and CD, we compute by Equations [4] to [7] the stresses in the middle and lower sections of these members (Figure 41).

For member AB:

the forces and moments in the middle section according to Equation [4] are:

$$m_e = m_1 + n_1 \frac{l_1}{2} - S_1 f_1 = 0.088 + 0.0336 \times \frac{1.62}{2} - 1.215 \times 0.146$$

$$= -0.062\,p\ [\text{t–m}]$$

$$n_e = n_1 = 0.0336\,p\,[\text{t}]$$

the forces and moments in the lower section are computed by Equation [5] as:

$$m_3 = m_1 + n_1 l_1 = 0.088 + 0.0336 \times 1.62 = 0.143\,p\,[\text{t–m}]$$

$$n_3 = n_1 = 0.0336\,p\,[\text{t–m}]$$

For member CD:
the forces and moments in the middle section are computed by Equation [6] as:

$$m_e' = m_2 - n_2 \times \frac{l_2}{2} + S_2 f_2 + \frac{l_2^2}{8} = 0.01818 - 0.817 \times \frac{1.55}{2} + 0.93 \times 0.925 + \frac{1.55^2}{8}$$

$$= -0.067\ p\ [\text{t-m}]$$

$$n_e' = n_2 - \frac{1}{2} l_2 = 0.817 - \frac{1.55}{2} = 0.042\ p\ [\text{t}]$$

the forces and moments in the lower section are computed by Equation [7] as:

$$m_4 = m_2 - n_2 l_2 + \frac{l_2^2}{2} = 0.01808 - 0.817 \times 1.55 + \frac{1}{2} \times 1.55^2 = 0.115\ p\ [\text{t-m}]$$

$$n_4 = n_2 - l_2 = 0.817 - 1.55 = -0.733\ p\ [\text{t}]$$

To obtain the numerical values of forces and moments, the intensity of the design load acting on the frame must be substituted for the symbol p.

For example, with a design load equal to 9 atm (design depth 90 [m]), and the distances between the frames equal to 0.5 [m], the intensity of the design load p = 0.5 x 90 = 45 [t] per running meter.

The axial tensile force in member CD is :

$$S_2 = 0.93\,p = 0.93 \times 45 = 41.8\ [\text{t}]$$

The bending moment in the lower section of member CD is :

$$m_4 = 0.115 \times 45 = 5.18\ [\text{t-m}]$$

The shearing force in the lower section of member CD is:

$$n_4 = 0.733\,p = 0.733 \times 45 = 33\ [\text{t}]$$

The maximum tensile stress in the lower section of member CD (at the IS edge of the section) is:

$$\sigma = \frac{S_2}{F} + \frac{m_4}{W}$$

where F is the area of the section and W is the moment of resistance of the section with respect to the inner edge.

Example 2.

To find the forces and moments in the sections of members of a frame without cross-stays (Figure 44), representing part of a composite frame as shown in Figure 14.

radius of the circumference of the frame, $r = 2.25$ [m]

radius of the inner member, $R = 2.25$ [m]

$$l_1 = \overline{AB} = 1.69 \text{ [m]} \quad l_2 = \overline{CD} = 1.69 \text{ [m]} \quad h_1 = \overline{AK} = 1.19 \text{ [m]}$$

$$h_2 = \overline{BK} = 1.21 \text{ [m]} \quad a = \overline{CL} = 0.9 \text{ [m]} \quad b = \overline{BL} = 0.375 \text{ [m]}$$

$$s = \overline{OC} = 1.69 \text{ [m]} \quad \overline{OF} = 0 \quad \overline{CF} = 1.69 \text{ [m]}$$

initial deflection of members AB and CD:

$$f_1 = \frac{l_1^2}{8r} = \frac{1.69^2}{8 \times 2.25} = 0.159 \text{ [m]}$$

$$f_2 = \frac{l_2^2}{8R} = \frac{1.69^2}{8 \times 2.25} = 0.159 \text{ [m]}$$

the direction of both deflections is positive;

the intensity of the load acting on the frame is p [t-m] (the direction of the load is positive);

the moments of inertia of the sections of the members are identical, $I_1 = I_2$.

We determine the external moments and forces reduced to joint C by Equations [1] to [3]:

$$\mathfrak{M} = \frac{1}{2} p(r^2 - s^2) = \frac{1}{2} p(2.25^2 - 1.69^2) = 1.103 p \text{ [t-m]}$$

$$N = p \times \overline{CF} = 1.69 p \text{ [t]}$$

$$S_0 = p \times \overline{OF} = 0$$

The moments and forces found have positive direction.

We next compute the numerical values for the quantities:

$$\sin\alpha = \frac{h_1}{l_1} = \frac{1.19}{1.69} = 0.704 \qquad \cos\alpha = \frac{h_2}{l_1} = \frac{1.21}{1.69} = 0.716$$

$$\mu_1 = \frac{2}{3} f_1 = \frac{2}{3} \times 0.159 = 0.106 \qquad \mu_2 = \frac{2}{3} f_2 = \frac{2}{3} \times 0.159 = 0.106$$

$$t = \frac{h_1}{a} = \frac{1.19}{0.9} = 1.322 \qquad \gamma = \frac{l_2}{l_1} = \frac{1.69}{1.69} = 1$$

$$k = t\gamma = 1.322 \qquad \chi = \gamma^2 \frac{l_2}{l_1} = 1$$

$$\eta = \frac{b}{a} \sin\alpha + \cos\alpha = \frac{0.375}{0.9} \times 0.704 + 0.716 = 1.0092$$

$$A = \frac{2(1.5 + k)}{3(2 + k)}\, l_2 = \frac{2(1.5 + 1.322)}{3(2 + 1.322)}\, 1.69 = 0.956$$

$$B = \frac{1.333 + k}{4(2 + k)}\, l_2^2 = \frac{1.333 + 1.322}{4(2 + 1.322)}\, 1.69^2 = 0.572$$

$$C = (2t + 3\eta)\chi = (2 \times 1.322 + 3 \times 1.009) \times 1 = 5.772$$

$$D = \frac{3(t + 2\eta)}{(2t + 3\eta)\, l_1} = \frac{3(1.322 + 2 \times 1.009)}{(2 \times 1.322 + 3 \times 1.009)\, 1.69} = 1.027$$

$$T = \mathfrak{M} + AN + \mu_2 S_0 - B = 1.103 + 0.956 \times 1.69 - 0.572 = 2.146\ ^*$$

$$K = a + A \sin\alpha + \mu_2 \cos\alpha + \mu_1 = 0.9 + 0.956 \times 0.704 + 0.106 \times 0.716 + 0.106 = 1.7549$$

$$L = b - A \cos\alpha + \mu_2 \sin\alpha + \frac{1}{D} = 0.375 - 0.956 \times 0.716 + 0.106 \times 0.704 + \frac{1}{1.027} = 0.7386$$

We compute the magnitudes of the forces and moments in first approximation by Equation [8] as

$$n_1 = 0$$

$$S_1 = \frac{T}{K} = \frac{2.146}{1.755} = 1.223$$

$$S_2 = S_1 \cos\alpha - S_0 = 1.223 \times 0.716 - 0 = 0.876$$

*Editor's Note. In the original, one term of the numerical computation was apparently omitted.

$$n_2 = N - S_1 \sin\alpha = 1.69 - 1.228 \times 0.704 = 0.828$$

$$m_2 = An_2 - S_2\mu_2 - B = 0.956 \times 0.828 - 0.876 \times 0.106 - 0.572 = 0.1272$$

$$m_1 = S_1\mu_1 + C\left[m_2 + \frac{2}{3}l_2n_2 + S_2\mu_2 + \frac{1}{4}l_2^2\right] = 1.228 \times 0.106 + 5.771\left[0.1272 - \frac{2}{3} \times 1.69 \times 0.828 + 0.876 \times 0.106 + 0.25 \times 1.69^2\right] = 0.1418$$

The values obtained are entered into the first column of Table 15. The corrections are then computed by Equations [9] and [10]:

$$\Delta_1 n_1 = D(S_1\mu_1 - m_1) = 1.027(1.228 \times 0.106 - 0.1418) = -0.01243$$

$$\Delta_1 S_1 = \frac{L}{K}\Delta_1 n_1 = \frac{0.7386}{1.7549}(-0.01243) = -0.00523$$

$$\Delta_1 S_2 = \Delta_1 S_1 \cos\alpha - \Delta_1 n_1 \sin\alpha = -0.00523 \times 0.716 + 0.01243 \times 0.704 = 0.005015$$

$$\Delta_1 n_2 = 0.00523 \times 0.704 + 0.01243 \times 0.716 = 0.01258$$

$$\Delta_1 m_2 = A\Delta_1 n_2 - \mu_2\Delta_1 S_2 = 0.956 \times 0.01258 - 0.106 \times 0.005045 = 0.01150$$

$$\Delta_1 m_1 = \mu_1\Delta_1 S_1 + C\left[\Delta_1 m_2 - \frac{2}{3}l_2\,\Delta_1 n_2 + \mu_2\Delta_1 S_2\right] = 0.106 \times 0.00523 + 5.771\left[0.01150 - \frac{2}{3} \times 1.69 \times 0.01258 + 0.106 \times 0.005015\right] = -0.01288$$

The values obtained are entered in the second column of Table 15; **the second correction $\Delta_2 n_1$ is calculated by Equation [11] and the coefficient** of total corrections μ by Equation [12],

$$\Delta_2 n_1 = 1.027\,(-0.00523 \times 0.106 + 0.01288) = 0.01267$$

$$\mu = \frac{-0.01243}{-0.01243 - 0.01267} = 0.495$$

Thereupon, the values in the second column are multiplied by $\mu = 0.495$ and the results are entered into the third column of Table 15. Thereupon, we find all the forces and moments indicated in Column IV of the

table.

The degree of correctness of the values found for the forces and moments is checked by Equation [13], namely:

$$S_1 a - \mathfrak{M} - m_2 + m_1 - n_1 b = 1.22 \times 0.9 - 1.103 - 0.133 + 0.135 + 0.00616 \times 0.375 = 1.237 - 1.236$$

Having found the forces and moments in the upper sections of members AB and CD, we compute the forces and moments in the middle and lower sections of these members by Equations [4] to [7]:

For member AB:

the forces and moments in the middle section, by Equation [4] are:

$$m_e = 0.13542 - 0.00616\left(\frac{1.69}{2}\right) - 1.22041 \times 0.159 = 0.064\ p\ [\text{t-m}]$$

$$n_e = n_1 = -0.0062\ p\ [\text{t}]$$

the forces and moments in the lower section by Equation [5] are:

$$m_3 = 0.13542 - 0.00616 \times 1.69 = 0.125\ p\ [\text{t-m}]$$

$$n_3 = n_1 = -0.0062\ p[\text{t}]$$

For member CD:

the forces and moments in the middle section by Equation [6] are:

$$m_e' = 0.1329 - 0.8342\ \frac{1.69}{2} + 0.8785 \times 0.159 + \frac{1.69^2}{8} = -0.075\ p\ [\text{t-m}]$$

$$n_e' = 0.834 - \frac{1.69}{2} = -0.011\ p\ [\text{t}]$$

the forces and moments in the lower section, by Equation [7] are:

$$m_4 = 0.133 - 0.834 \times 1.69 + \frac{2.86}{2} = 0.153\ p\ [\text{t-m}]$$

$$n_4 = n_2 - l_2 = 0.834 - 1.69 = -0.856\ p\ [\text{t}]$$

To find the numerical values for the forces and moments, the intensity of the design load acting on the frame must be substituted for p.

<u>Example 3.</u>

To find the forces and moments in the sections of the members of a frame without cross-stays (Figure 45), as shown in Figure 31, representing part of a composite frame shown in Figure 17.

radius of the circumference of the frame, $r = 2.68$ [m]

radius of the loaded member CD, $R = 2.73$ [m]

$$l_1 = \overline{AB} = 0.6 \ [m] \quad l_2 = \overline{CD} = 1.01 \ [m] \quad h_1 = \overline{AK} = 0.04 \ [m]$$
$$h_2 = \overline{BK} = 3.6 \ [m] \quad a = \overline{CL} = 1.24 \ [m] \quad b = \overline{BL} = 0.275 \ [m]$$

the initial deflection of member AB is:

$$f_1 = \frac{l_1^2}{8r} = \frac{0.6^2}{8 \times 2.68} = 0.0168 \ [m]$$

the same magnitude for member CD is:

$$f_2 = \frac{l_2^2}{8R} = \frac{1.01^2}{8 \times 2.73} = 0.0467 \ [m]$$

the direction of the deflection f_1 is negative, therefore, the minus sign must be used.

The intensity of the load acting on the frame is p [t m^{-1}] (the direction of the load on member CD is negative; therefore, according to what was said in Section 15, Paragraph 3(i), the minus sign must be used).

The external loading acting on the frame is equivalent to one concentrated force P = ps, normal to the radius drawn from the center O to the point C and distant from C by

$$e = \overline{CE} = \frac{1}{2} s \, (r^2 - s^2)$$

where s is equal to the length of the segment $\overline{OC}$ (s = 3.965 [m]).

Reducing this force to point C and resolving it in the direction of the member CD and perpendicular to it, we get (instead of the decomposition of force P, we resolve the segment CD):

the bending moment by Equation [1] is:

$$\mathfrak{M} = Pe = \frac{1}{2} p \, (r^2 - s^2) = 4.27 \, p \ [t\text{-}m]$$

the force in the direction of member CD by Equation [2] is:

$$S_0 = \overline{OF} \times p = 3.95 \, p \ [t]$$

the force normal to the member by Equation [3] is:

$$N = \overline{CF} \times p = 0.238 \, p \ [t]$$

as the force N has negative direction, it must be provided with the minus sign.

We compute the numerical values of the quantities:

$$\sin\alpha = \frac{h_1}{l_1} = \frac{0.04}{0.6} = 0.0667 \qquad \cos\alpha = \frac{h_2}{l_1} = \frac{0.6}{0.6} = 1$$

$$\mu_1 = \frac{2}{3} f_1 = -0.667 \times 0.0168 = -0.0112$$

$$\mu_2 = 2/3\, f_2 = 0.667 \times 0.0467 = 0.0312$$

$$t = \frac{h_1}{a} = \frac{0.04}{1.24} = 0.0322$$

$$\gamma = \frac{l_2}{l_1} = \frac{1.01}{0.6} = 1.68$$

$$k = t\gamma = 0.0542$$

$$\chi = \gamma^2 I_1 / I_2 = 2.33$$

$$\eta = \frac{b}{a} \sin\alpha + \cos\alpha = 0.222 \times 0.0667 + 1 = 0.0148 + 1 = 1.0148$$

$$A = \frac{2(1.5 + k)}{3(2 + k)} l_2 = \frac{2(1.5 + 0.0542)}{3(2 + 0.0542)} 1.01 = 0.505$$

$$B = \frac{1.333 + k}{4(2 + k)} l_2^2 = \frac{1.387}{4(2 + 0.0542)} 1.02 = 0.172$$

$$C = (2t + 3\eta)\chi = (0.0644 + 3 \times 1.0148)\, 2.83 = 3.1088 \times 2.83 = 8.81$$

$$D = \frac{3(t + 2\eta)}{(2t + 3\eta)\, l_1} = \frac{3(0.0522 + 2.0296)}{3.1088 \times 0.60} = 3.32$$

$$T = \mathfrak{M} + AN + \mu_2 S_0 + \mu_1 = 4.27 - 0.238 \times 0.505 + 0.0312 \times 3.95 + 0.172 = 4.445$$

$$K = a + A \sin\alpha + \mu_2 \cos\alpha + \mu_1 = 1.2932$$

$$L = b - A \cos\alpha + \mu_2 \sin\alpha + \frac{1}{D} = 0.0691$$

By Equation [8], we compute the values of the moments and forces:

$$n_1 = 0$$

$$S_1 = \frac{T}{K} = \frac{4.448}{1.2932} = 3.44$$

$$S_2 = S_1 \cos\alpha - S_0 = 3.44 \times 1 - 3.95 = -0.51$$

$$n_2 = N - S_1 \sin\alpha = -0.238 - 3.44 \times 0.0667 = -0.468$$

$$m_2 = AN_2 - S_2\mu_2 + B = -0.505 \times 0.468 + 0.51 \times 0.0312 + 0.172 = -0.048$$

$$m_1 = -3.44 \times 0.0112 + 8.81\left[-0.048 + 0.067 \times 1.01 \times 0.468 - 0.51 \times 0.0312 - \frac{1.02}{4}\right] = -0.0729$$

We compute the corrections to the first approximation by Equations [9] and [10] as:

$$\Delta_1 n_1 = 3.32(-3.44 \times 0.0112 + 0.729) = 0.114$$

$$\Delta_1 S_1 = \frac{L}{K}\Delta_1 n_1 = \frac{0.0691}{1.2932}(0.114) = 0.00608$$

$$\Delta_1 S_2 = 0.00608 - 0.114 \times 0.0667 = -0.00152$$

$$\Delta_1 n_2 = -0.00608 \times 0.7667 - 0.114 = -0.114^*$$

$$\Delta_1 m_2 = -0.505 \times 0.114 + 0.00152 \times 0.0312 = -0.0575$$

$$\Delta_1 m_1 = -0.0112 \times 0.00608 + 8.81(-0.0575 + 0.0767 - 0) = 0.169$$

We compute the second correction for the stress n_1 and the total correction as:

$$\Delta_2 n_1 = 3.32(-0.00608 \times 0.0112 - 0.169) = -0.562$$

$$\mu = \frac{0.114}{0.114 + 0.562} = \frac{0.114}{0.676} = 0.169$$

All the given calculations, as in the preceeding examples, are tabulated in Table 16.

The degree of accuracy is checked by Equation [13]:

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*Editor's Note: Should be −0.1187.

$$S_1 a - \mathfrak{M} - m_2 + m_1 - n_1 b = 0.058 - 0.049$$

The forces and moments in the central and lower sections of the members are computed by Equations [4] to [7].

16. DERIVATION OF EXPRESSIONS FOR THE FORCES AND MOMENTS IN THE MEMBERS OF A FRAME WITHOUT CROSS-STAYS (Figure 39).

The lower rigid member of the frame AD will be considered immovable and all the external forces acting on the upper rigid member BC will be reduced to the upper joint C of the loaded member CD, having decomposed the concentrated force in the direction of this member and normal to it.

The following notation will be used:

- $\mathfrak{M}$ is the external bending moment acting on member BC;
- S_0 is the force in the direction of member CD;
- N is the force in the direction normal to CD;
- p is the intensity of the uniformly distributed load acting on CD;
- m_1, n_1, S_1 are, respectively, the bending moment, resultant force and axial force acting in the upper section of member AB;
- m_2, n_2, S_2 are the same for the upper section of member CD.

For positive directions of $\mathfrak{M}$, S_0, N and p, we shall take their directions as shown in Figure 39, and, for the positive directions of the moments and forces m_1, n_1 and S_1 and for m_2, n_2, S_2, those indicated in Figure 41.

The remaining conventional symbols and relations, entering in further expressions correspond completely to those adopted in Section 15, Paragraph 3, for investigation of analysis of girders without cross-stays.

The frame without cross-stays under investigation, contains six unknown forces and moments: S_1, S_2, m_1, m_2, n_1, n_2, and consequently represents a statically indeterminate system with three unknowns.

Projecting all the forces acting on member BC in the direction of member CD and the direction normal to it, and setting up the moment equation of all forces reduced to point C (Figure 46), the following three relationships between these forces may be written:

$$S_1 = \frac{1}{a}(\mathfrak{M} + m_2 - m_1 + n_1 b) \qquad [1]$$

$$S_2 = S_1 \cos\alpha - S_0 - n_1 \sin\alpha \qquad [2]$$

$$n_2 = N - S_1 \sin\alpha - n_1 \cos\alpha \qquad [3]$$

The remaining three relationships necessary for determination of all the unknown forces and moments will be found by application of the principle of least work, i.e. equating to zero the derivatives of the potential energy of the system with respect to each of the three redundant unknowns.

As redundant quantities, we shall take the moments m_1, m_2 and the force n_1. Directly from examination of Figure 41 the following expressions for the bending moments in AB and CD can be written -- assuming their initial shape a sine curve, i.e. according to:

$$y = f \sin\frac{\pi x}{l}$$

where f is the maximum deflection at the middle of the member and l is its length.

For member AB:

$$M_x = m_1 + n_1 x - S_1 f_1 \sin\frac{\pi x}{l}$$

For member CD:

$$M_{1x} = m_2 - n_2 x + S_2 f_2 \sin\frac{\pi x}{l_2} + \frac{px^2}{2}$$

The expression for the potential energy for members AB and CD is:

$$V = \frac{1}{2EI_1}\int_0^{l_1} M_x^2\,dx + \frac{1}{2EI_2}\int_0^{l_2} M_{1x}^2\,dx$$

The equations resulting from the principle of least work are:

$$\frac{\partial V}{\partial m_1} = \frac{1}{I_1}\int_0^{l_1} M_x \frac{\partial M_x}{\partial m_1}\,dx + \frac{1}{I_2}\int_0^{l_2} M_{1x}\frac{\partial M_{1x}}{\partial m_1}\,dx = 0 \qquad [a]$$

$$\frac{\partial V}{\partial n_1} = \frac{1}{I_1}\int_0^{l_1} M_x \frac{\partial M_x}{\partial n_1}\,dx + \frac{1}{I_2}\int_0^{l_2} M_{1x}\frac{\partial M_{1x}}{\partial n_1}\,dx = 0 \qquad [b]$$

$$\frac{\partial V}{\partial m_2} = \frac{1}{I_1}\int_0^{l_1} M_x \frac{\partial M_x}{\partial m_2}\,dx + \frac{1}{I_2}\int_0^{l_2} M_{1x}\frac{\partial M_{1x}}{\partial m_2}\,dx = 0 \qquad [c]$$

In evaluating the derivatives from the expressions for the bending moments with respect to the redundant unknowns m_1, n_1 and m_2, it is expedient to make an assumption which simplifies all the subsequent calculations, setting the derivatives of the axial forces S_1 and S_2 equal to 0; it is easy to see that such a simplifying assumption is equivalent to assuming that the displacements of the upper ends of members AB and CD, produced by forces S_1 and S_2 and by an initial bending in the members, are negligibly small in comparison with the displacements caused by the other stresses; consequently the less the curvature of AB and CD the more this assumption approximates reality.

Using the assumption made, the expressions for the derivatives entering into Equations [a], [b] and [c] may be stated as follows:

$$\frac{\partial M_x}{\partial m_1} = 1 \qquad \frac{\partial M_{1x}}{\partial m_1} = -\frac{\sin\alpha}{a}\,x = -\frac{h_1}{l_1 a}\,x = -t\,\frac{x}{l_1}$$

$$\frac{\partial N_x}{\partial n_1} = x \qquad \frac{\partial M_{1x}}{\partial n_1} = \left(\frac{b}{a}\sin\alpha + \cos\alpha\right) x = \eta x$$

$$\frac{\partial M_x}{\partial m_2} = 0 \qquad \frac{\partial M_{1x}}{\partial m_2} = 1 + \frac{\sin\alpha}{a}\,x = 1 + \frac{h_1}{a l_1} = 1 + t\,\frac{x}{l_1}$$

Substituting the values of the derivatives in Equation [a] and carrying out the integration, this equation will become:*

$$\frac{1}{l_1}\left[m_1 l_1 + \frac{m_1 l_1^2}{2} - S_1 f_1\,\frac{2l_1}{3}\right] - \frac{t}{l_2 l_1}\left[m_2\,\frac{l_2^2}{2} - n_1\,\frac{l_2^3}{3} + S_2' f_2\,\frac{l_2^2}{3} + \frac{p l_2^4}{4}\right] = 0$$

factoring out l_1 and l_2^2 and substituting the conventional symbols, we get:

---

*In the expressions for the integrals $\int_0^l \sin\frac{\pi x}{l} = \frac{2l}{\pi}$ and $\int_0^l x\sin\frac{\pi x}{l}\,dx = \frac{l^3}{\pi}$, we have taken $\pi = 3$, which is equivalent to assuming that the initial deflection of the member approaches more closely a circular arc.

$$m_1 + \frac{1}{2} n_1 l_1 - S_1\mu_1 - \frac{1}{2} t \chi \left(m_2 - \frac{2}{3} n_2 l_2 + S_2\mu_2 + \frac{p l_2^2}{4} \right) = 0 \qquad [d]$$

Substituting the values of the derivatives in Equations [b] and [c], just as in the foregoing, these equations become:

$$m_1 + \frac{2}{3} n_1 l_1 - S_1\mu_1 + \chi\eta \left[m_2 - \frac{2}{3} n_2 l_2 + S_2\mu_2 + \frac{p_2 l_2^2}{4} \right] = 0 \qquad [e]$$

$$m_2 (1 + 0.5k) - n_2 l_2 \left(0.5 + \frac{1}{3} k\right) + S_2\mu_2 (1 + 0.5k) + \frac{p l_2^2}{4} \left(\frac{2}{3} + 0.5k \right) = 0 \qquad [f]$$

Solving Equation [f] with respect to m_2 and introducing the conventional notation adopted, we shall have:

$$m_2 = A n_2 - S_2\mu_2 - Bp \qquad [4]$$

Multiplying Equation [d] by 4/3 and subtracting it from Equation [e], we get:

$$-\frac{1}{3} m_1 + \frac{1}{3} S_1\mu_1 + m_2 \left(\frac{2}{3} t + \eta \right) \chi - \frac{2}{3} n_2 \left(\frac{2}{3} t + \eta \right) \chi l_2$$

$$+ S_2\mu_2 \left(\frac{2}{3} t + \eta \right) \chi + \frac{p l_2^2}{4} \left(\frac{2}{3} t + \eta \right) \chi = 0$$

Solving the last equation with respect to m_1 and introducing the conventional notation adopted, we get:

$$m_1 = S_1\mu_1 + C \left[m_2 - \frac{2}{3} l_2 n_2 + S_2\mu_2 + \frac{p l_2^2}{4} \right] \qquad [5]$$

Multiplying Equation [d] by $2\eta/t$ and adding it to Equation [e], we get:

$$m_1 \left(2 \frac{\eta}{t} + 1\right) + n_1 l_1 \left(\frac{\eta}{t} + \frac{2}{3}\right) - S_1 \mu_1 \left(2 \frac{\eta}{t} + 1\right) = 0$$

Solving this equation for n_1 and introducing the conventional notation adopted, we have:

$$n_1 = D\,(S_1 \mu_1 - m_1). \qquad [6]$$

The Equations [4], [5] and [6] thus found, together with Equations [1], [2] and [3], obtained from the conditions of statics, give all six conditions required for determining all the unknown forces and moments in a frame without cross-stays.

The computation of these forces and moments for a given case may be made most conveniently by the following method.

Using Equations [2] to [6], the moments m_1 and m_2 will be expressed by the force n_1 and substituted in Equation [1] for S_1; solving the equation thus obtained for S_1, we shall find:

$$S_1 = \frac{T}{K} + \frac{L}{K}\, n_1, \qquad [7]$$

where

$$T = m + AN + \mu_2 S_0 \mp Bp,$$

$$K = a + A \sin \alpha + \mu_2 \cos \alpha + \mu_1,$$

$$L = b - A \cos \alpha + \mu_2 \sin \alpha + \frac{1}{D}\ .$$

Setting $n_1 = 0$ in first approximation, we can calculate, for the first approximation, the forces and moments according to Equations [7], and [2] to [5].

It is judicious to carry out all the calculations in a table, similar to Table 13 (Section 15). For this purpose, the forces and moments found in the first approximation are entered in Column I of the table.

Thereupon, we compute the correction to the first approximation for n_1 by Equation [6], i.e. $\Delta_1 n_1 = D(S_1\mu_1 - m_1)$. Further, we compute the corrections for the first approximation for the other forces and moments, as determined by correction $\Delta_1 n_1$ using the following expressions which result directly from Equations [7], and [2] to [5]:

$$\Delta_1 S_1 = \frac{L}{K} \Delta_1 n_1$$

$$\begin{aligned} &\Delta_1 S_2 = \Delta_1 S_1 \cos \alpha - \Delta_1 n_1 \sin \alpha \\ &\Delta_1 n_2 = - \Delta_1 S_1 \sin \alpha - \Delta_1 n_1 \cos \alpha \\ &\Delta_1 m_2 = A\Delta_1 n_2 - \Delta_1 S_1 \mu_2 \\ &\Delta_1 m_1 = \mu_1 \Delta_1 S_1 + C \left[\Delta_1 m_2 - \frac{2}{3} l_2 \Delta_1 n_2 + \mu_2 \Delta_1 S_2 \right] \end{aligned} \qquad [8]$$

The corrections obtained are entered in Column II of the table.

Next, we calculate the following correction for the force n_1, resulting directly from Equation [6], i.e.

$$\Delta_2 n_1 = D(\Delta_1 S_1 \mu_1 - \Delta_1 m_1). \qquad [9]$$

Making the computations of the subsequent corrections for the desired forces and moments, we could finally find them with the necessary degree of accuracy. However, this objective can be attained more quickly and with satisfactory accuracy by using a rectilinear interpolation for finding the values of the forces and moments corresponding to a zero value of the correction for the force n_1.

It is easy to show that the total corrections for the first approximation (i.e. with respect to Column I of the table) may be found by multiplying the values in Column II of the table by a common multiplier equal to

$$\mu = \frac{\Delta_1 n_1}{\Delta_1 n_1 - \Delta_2 n_1} \qquad [10]$$

Entering the results of this multiplication in Column III of the table, and adding them to the values of I and III, we shall get in Column IV the desired values for all the forces and moments.

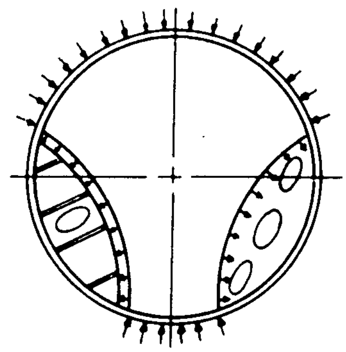


Figure 14

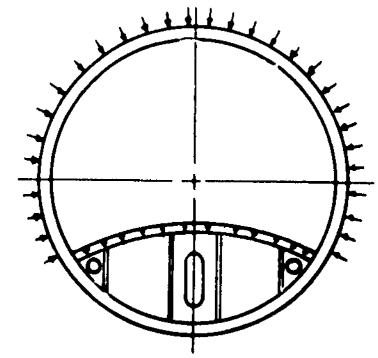


Figure 15

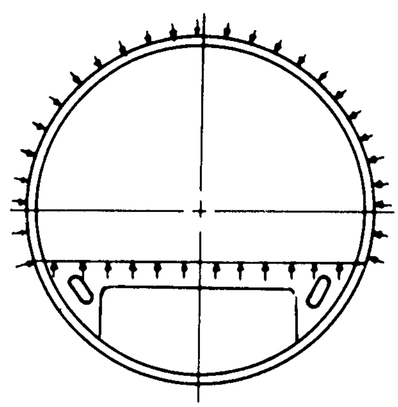


Figure 16

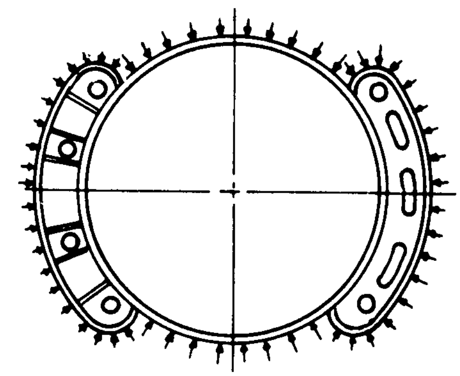


Figure 17

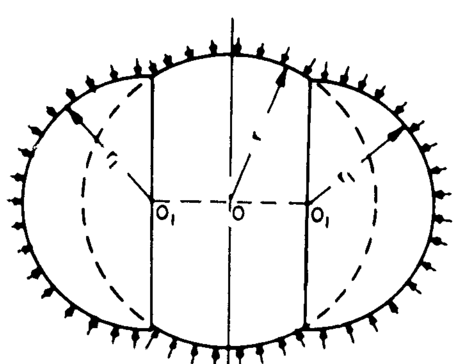
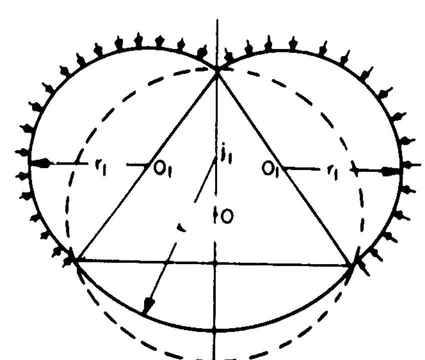


Figure 18



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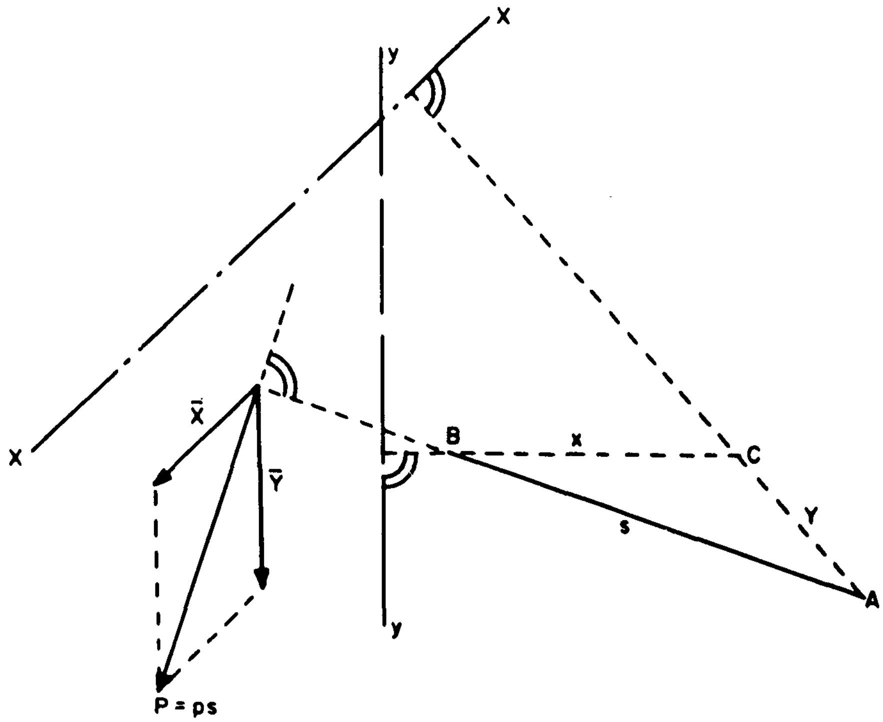

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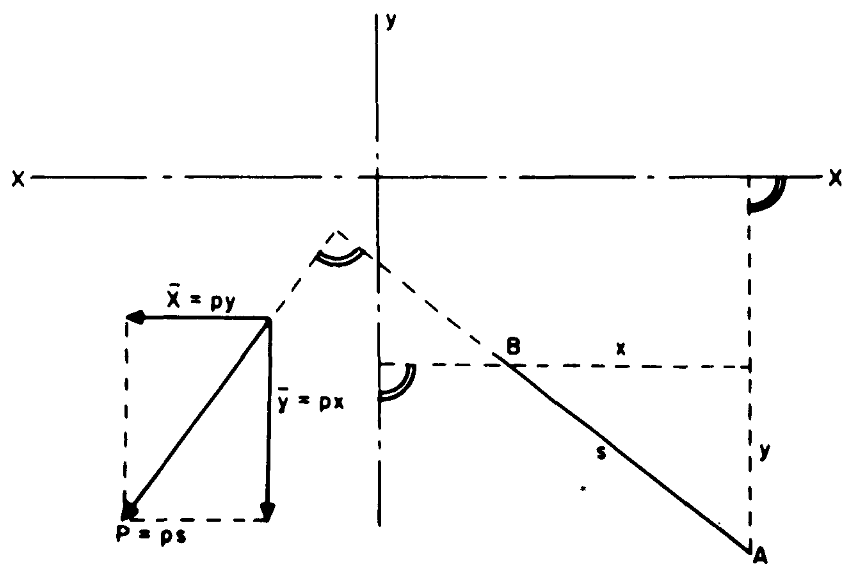
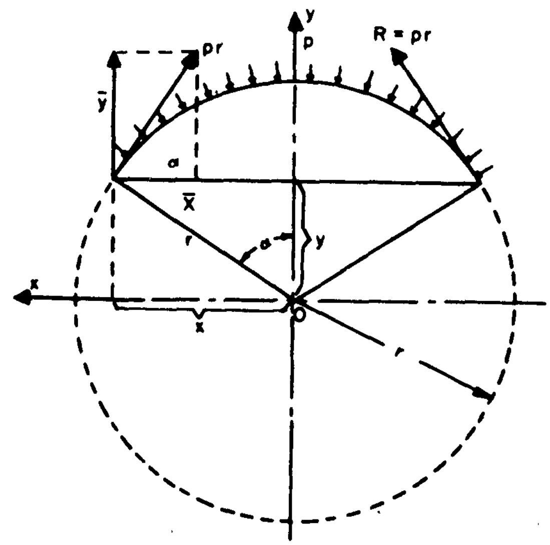


Figure 21



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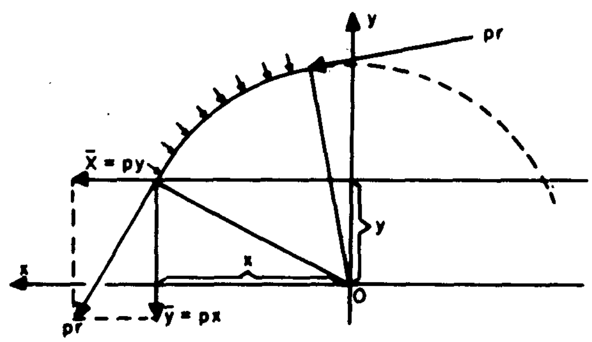

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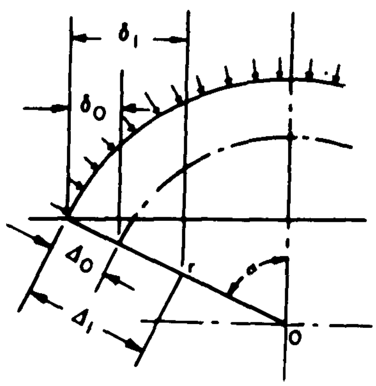

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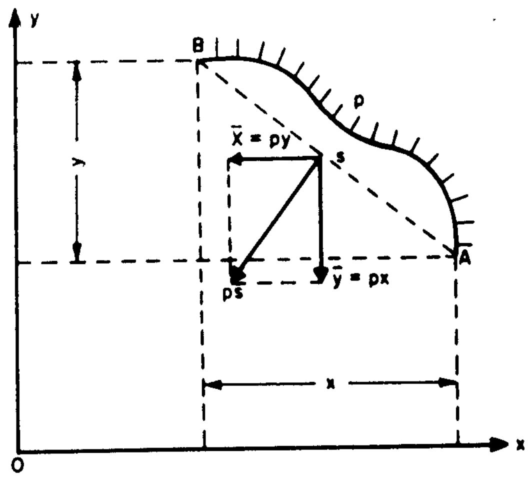

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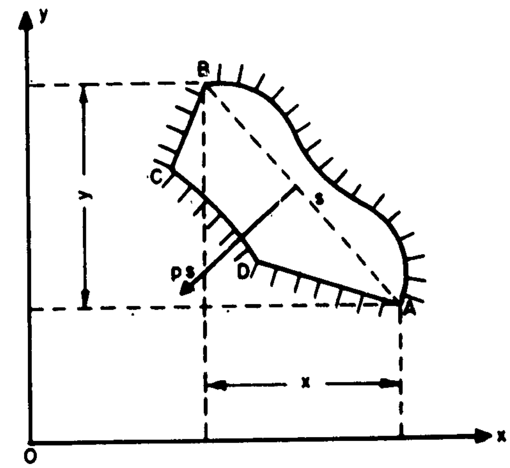

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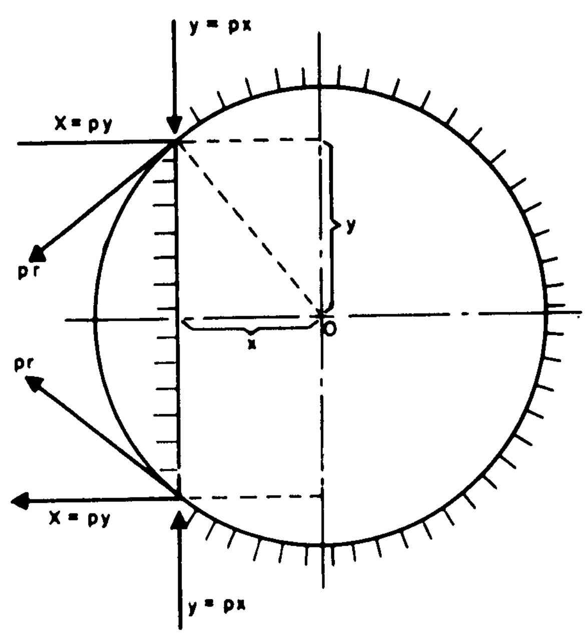
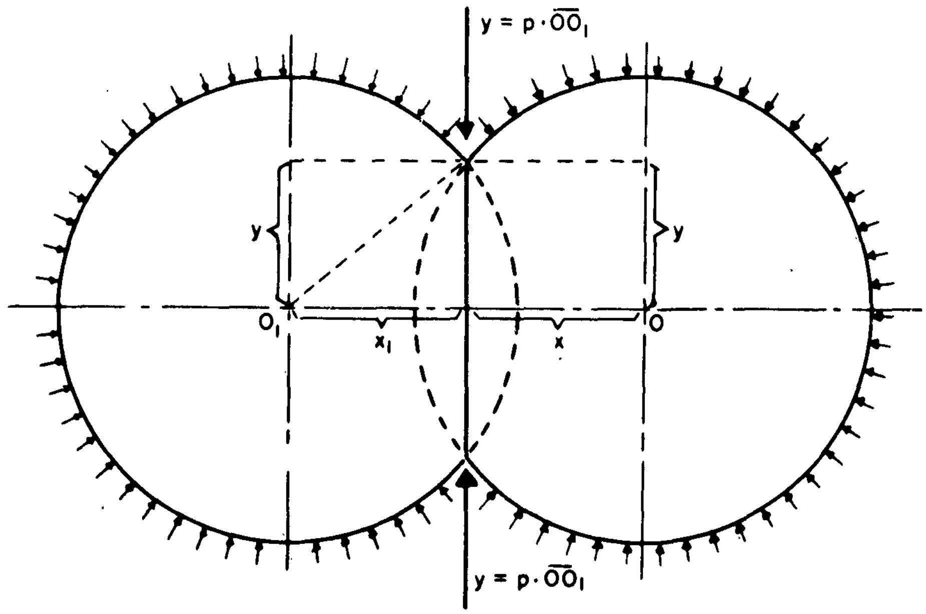


Figure 27



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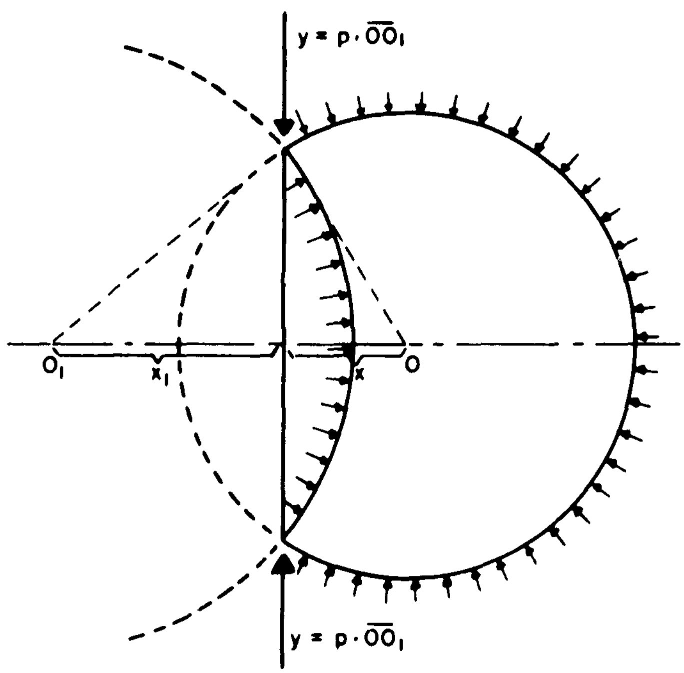

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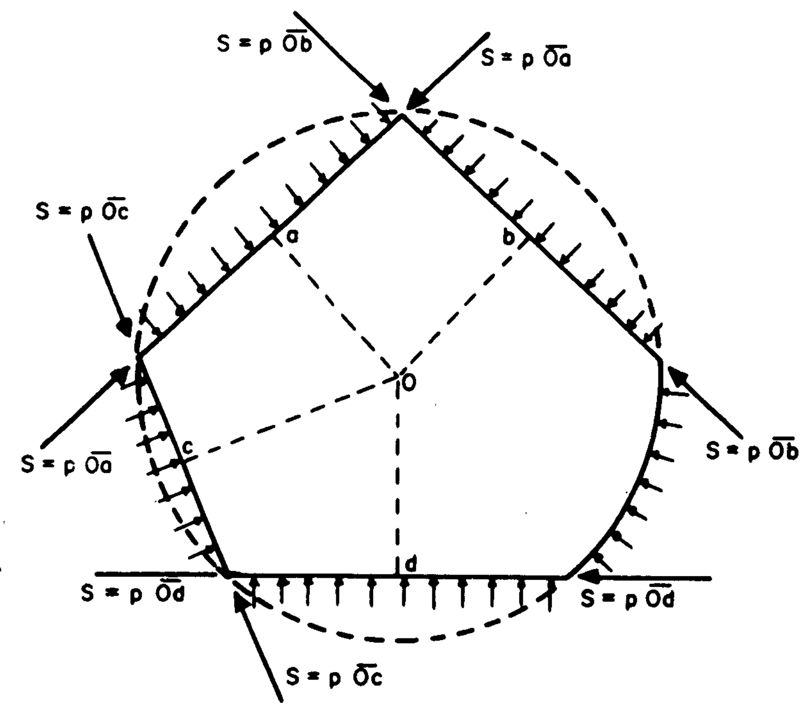
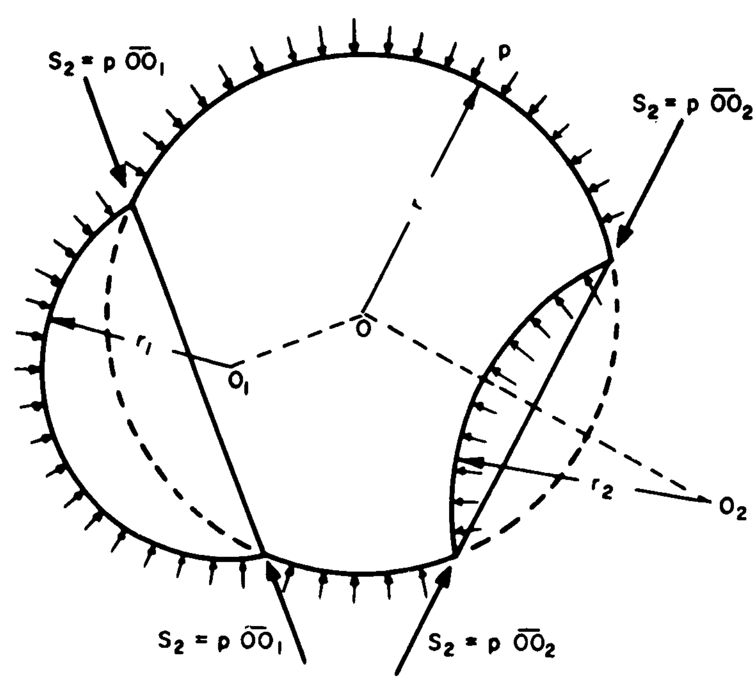


Figure 30



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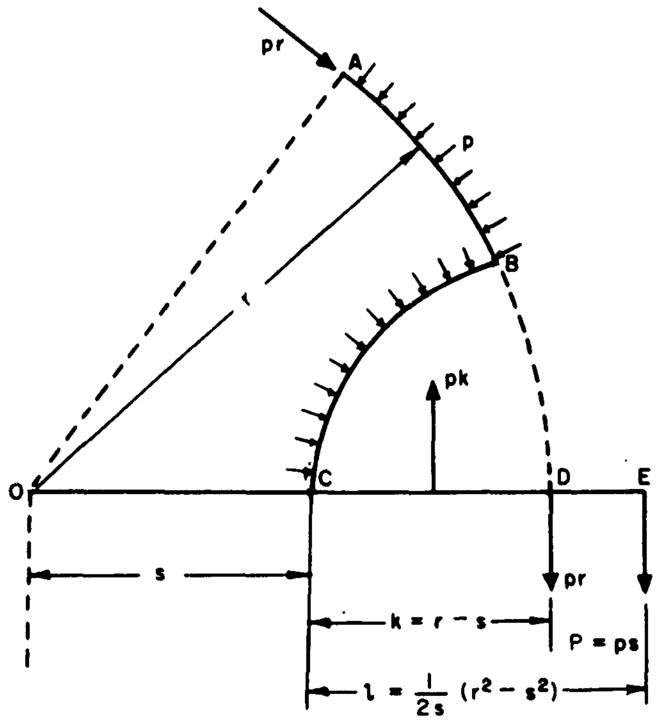

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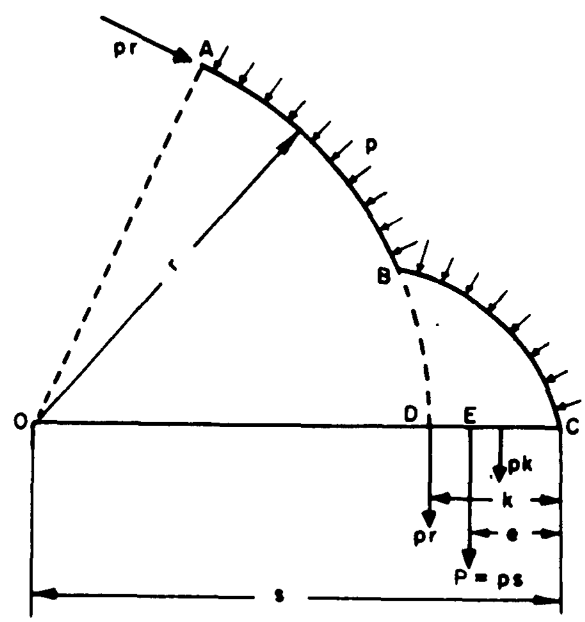


Figure 33

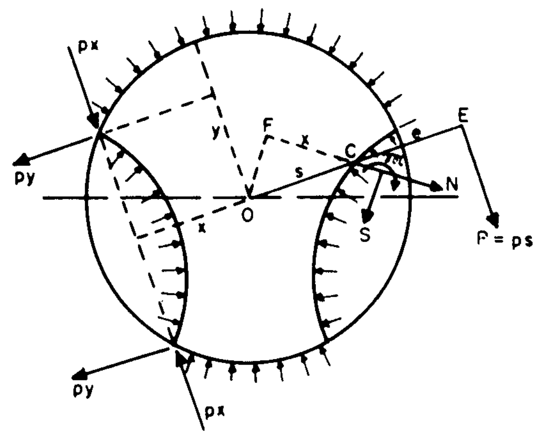


Figure 34

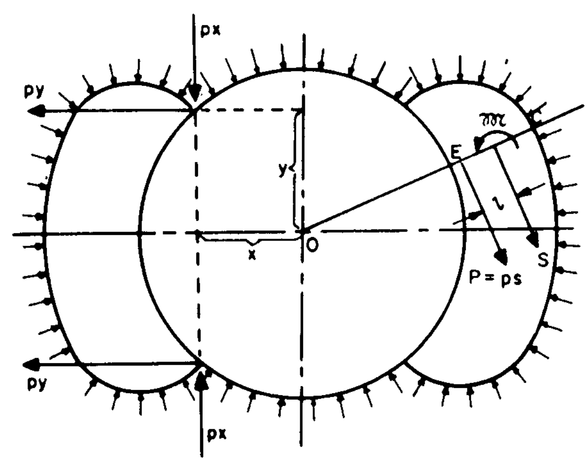


Figure 35

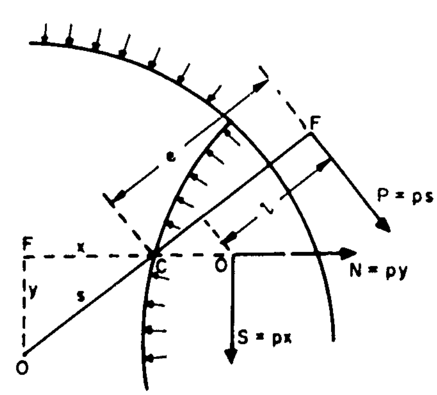


Figure 36

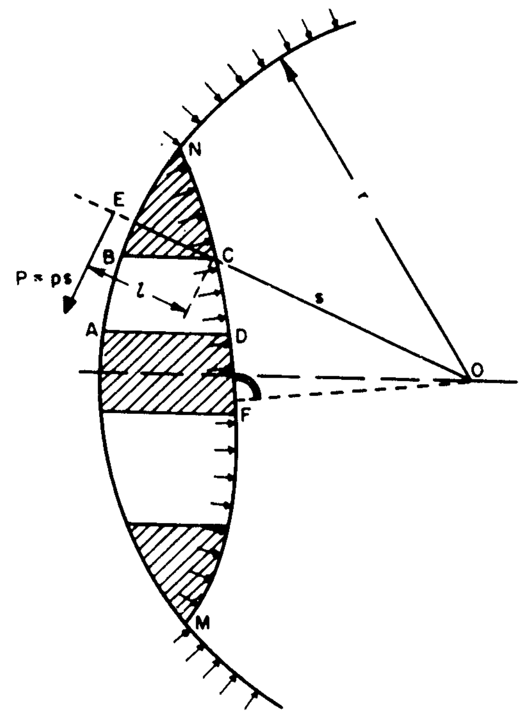


Figure 37

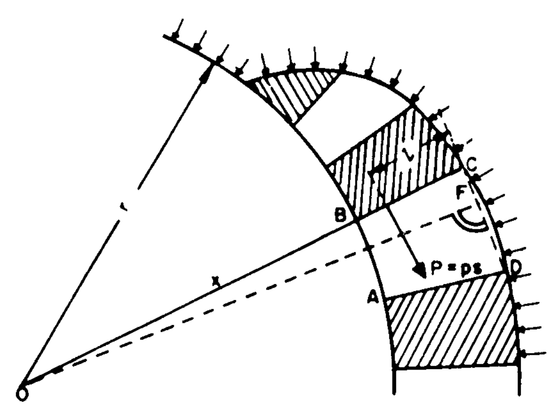
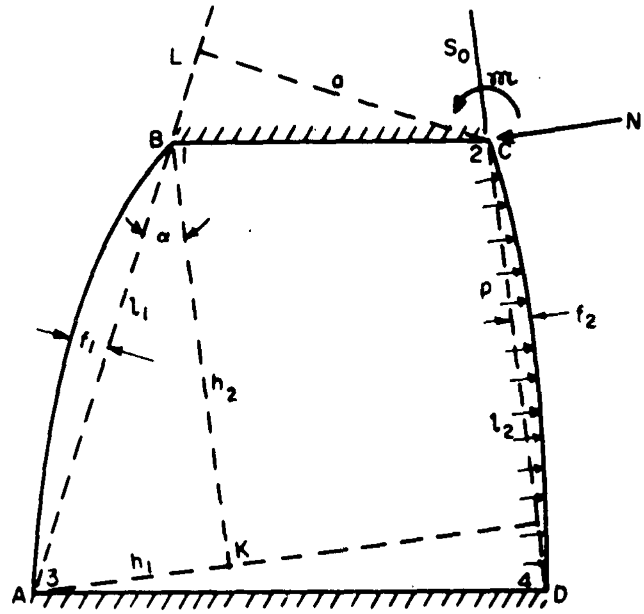


Figure 38



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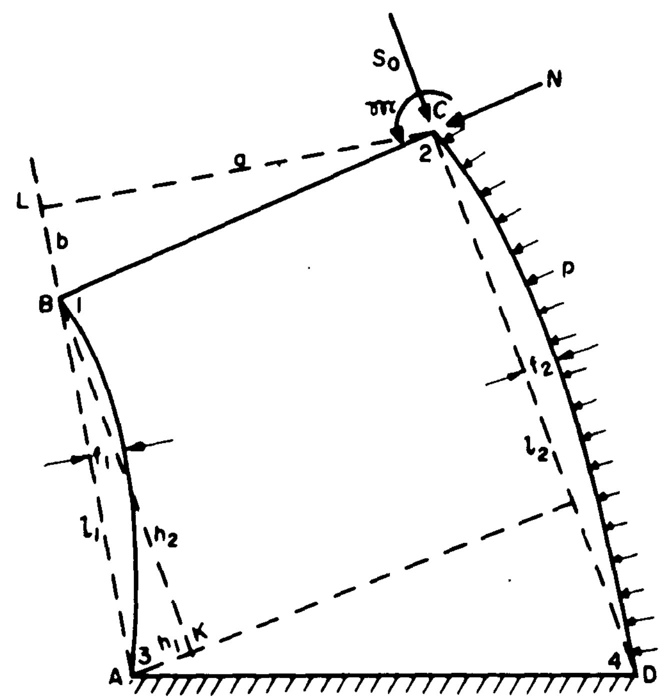

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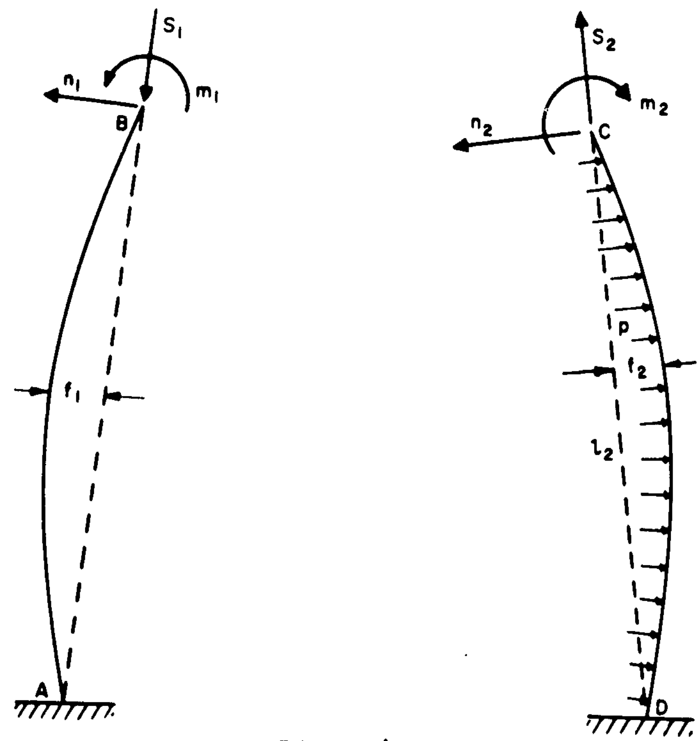

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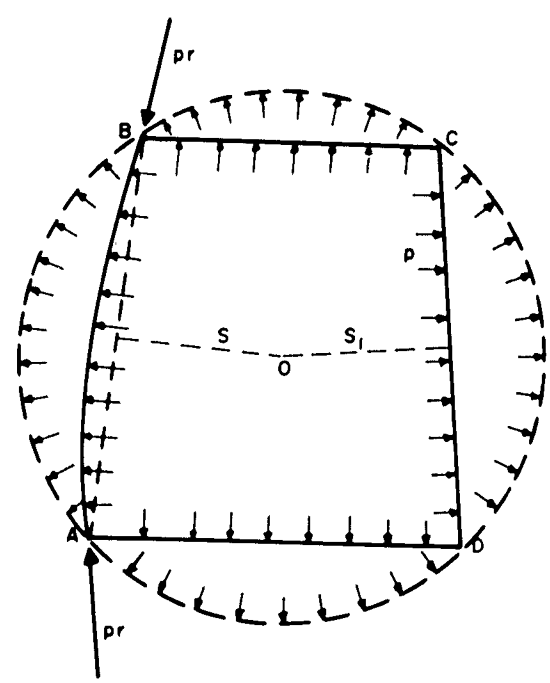

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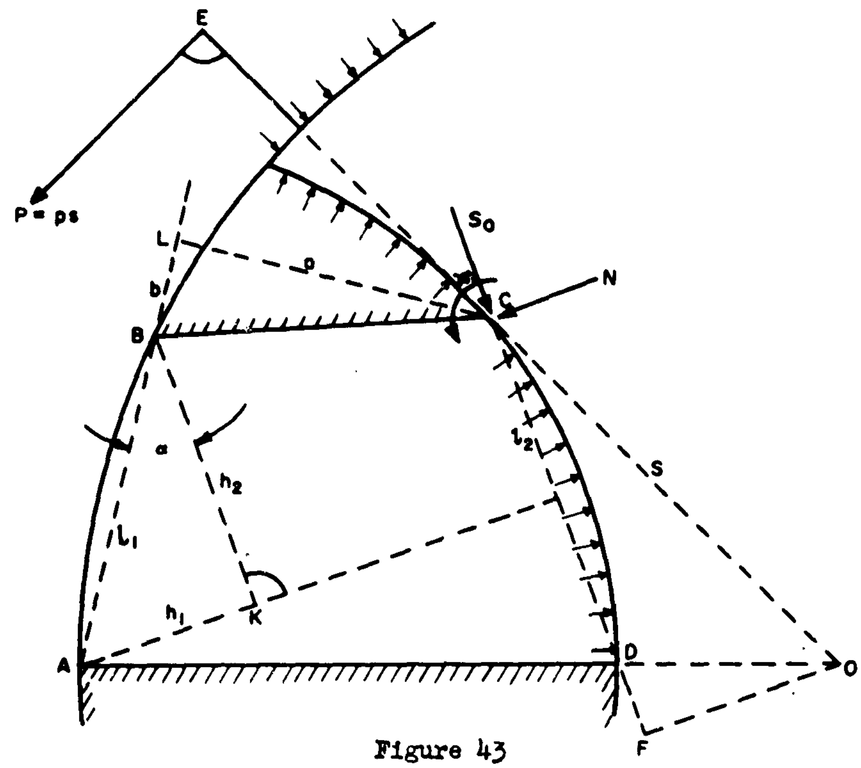


Figure 43

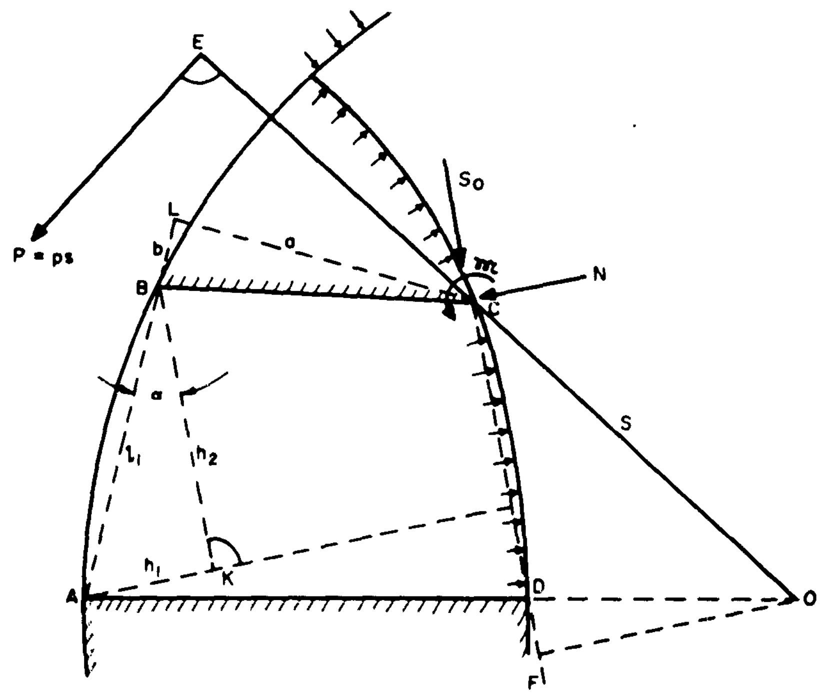
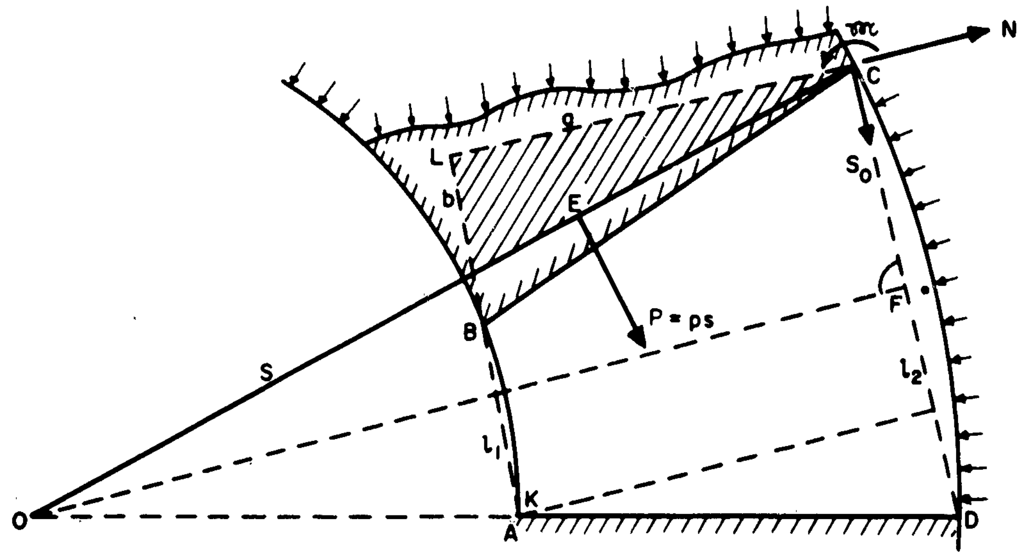


Figure 44



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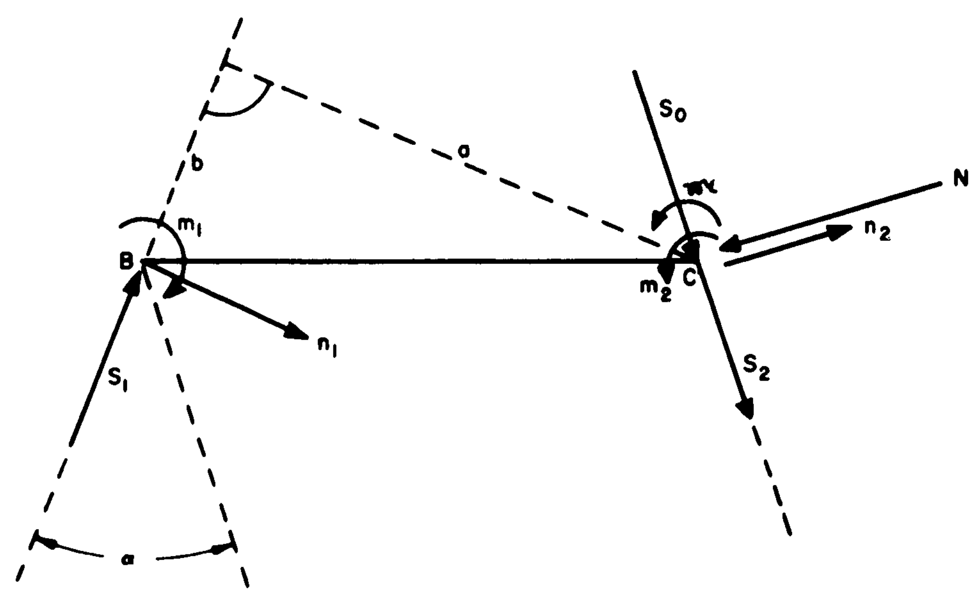

Figure 46

TABLE 13

Stresses and Moments	I	II	III	IV
	By Equation (8)	By Equations (9) & (10)	$\mu \cdot$ II	I + III
n_1	0	$\Delta_1 n_1$	$\mu\Delta_1 n_1$	$0 + \mu\Delta_1 n_1$
S_1				
S_2				
n_2				
m_2				
m_1				

TABLE 14

Stresses and Moments	I	II	III	IV
	By Equation (8)	By Equations (9) & (10)	$\mu \cdot$ II	I + III
n_1	0	0.0652	0.0336	0.0336
S_1	1.202	0.0245	0.0126	1.215
S_2	0.940	0.0195	−0.0101	0.930
n_2	0.852	0.0680	−0.0350	0.817
m_2	0.210	0.0566	−0.0292	0.181
m_1	0.059	0.0571	0.0294	0.088

TABLE 15

Stresses and Moments	I	II	III	IV
	By Equation (8)	By Equations (9) & (10)	$\mu \cdot$ II	I + III
n_1	–	−0.01243	−0.00616	−0.00616
S_1	1.223	−0.00523	−0.00259	−1.22041
S_2	0.876	0.00501	−0.00246	0.87848
n_2	0.828	0.01258	0.00617	0.83417
m_2	0.1272	0.01150	0.00570	0.13290
m_1	0.1418	−0.01288	−0.00638	0.13542

TABLE 16

Stresses and Moments	I	II	III	IV
	By Equation (8)	By Equations (9) & (10)	$\mu \cdot$ II	I + III
n_1	0	0.114	0.0193	0.019
S_1	3.44	0.00608	0.0011	3.441
S_2	−0.51	−0.00152	−0.0003	−0.510
n_2	−0.468	−0.114	−0.0197	−0.488
m_2	−0.048	−0.0575	−0.0099	−0.058
m_1	−0.0729	0.169	0.0292	−0.043

CHAPTER V

STRENGTH ANALYSIS OF BULKHEADS

17. VARIOUS TYPES OF BULKHEADS

1. Submarine bulkheads may be divided into the following types according to their purpose and the strength requirements corresponding to this purpose.

(a) Terminal bulkheads, i.e. bulkheads limiting the pressure hull of submarines. These bulkheads must have the same strength as the pressure hull. Therefore, their strength must be calculated for the design depth of submergence, determined by Equation [9], Section 3, Part 1, permitting at such pressure the increase of tensile stresses in them to a magnitude of 25% above the yield point. The compressive stress must not exceed a magnitude corresponding to the loss of general and local stability of connections of the bulkhead.

(b) Bulkheads bounding the heavy-duty tanks of the hull (high-speed diving tanks, fuel and ballast tanks, located within the pressure hull* etc.). Bulkheads of this type must withstand the same pressure as the entire pressure hull, in addition to the possibility of a certain pressure increase when blowing down the tanks by high pressure compressed air at the limit depth of submersion.

(c) Light water-tight bulkheads within the pressure hull, designed to prevent the submarine from sinking in case of damage to the pressure hull when surfaced. The strength of such bulkheads must be calculated for the pressure of a column of water higher than the upper edge of the bulkhead for a number of the order of 5 meters, with allowable stresses equal to those usually taken for calculation of the strength of emergency bulkheads of surface vessels.

(d) Heavy-duty bulkheads within the pressure hull, designed not only to prevent the submarine from sinking when damaged during surface navigation, but also to produce water-tight compartments within the pressure hull which will stand up at great depths of submersion. At the design pressure, which is fixed by the structural specifications for submarine construction, the total stresses in the plates of the bulkhead plating must not exceed the yield point of the plating material and in the joints of the structure 80% of the yield point of the material.

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*Editor's Note: The description of submarines containing inboard ballast, trim and diving tanks appears to refer to the type of submarines formerly built by the French and British Navies, rather than those built by the German and U.S. Navies where the ballast-fuel tanks were all situated in the outer hull.

(e) Water-tight bulkheads of the outer hull (outside ballast and fuel tanks, etc.). The design load for such bulkheads, must be taken as the greatest pressure exerted on the bulkhead produced under operating conditions of the submarine. The allowable stresses in the bulkhead connections must be determined on the basis of the character of the applied load in conformity with the usual standards for hull construction of surface vessels.

The rules given above for strength analysis of bulkheads, apply not only to bulkheads but also to the calculation of other partitions which bound the solid or light tanks of submarine hulls. When specifying standards of allowable stresses for the plating and construction of such partitions, the character of the load acting on them must be considered. Thereby, it must be borne in mind that, in the partitions which can repeatedly be subjected to pressure from opposite sides, the stresses in the plating and even more so, in the connections of the elements of the partition, must not exceed the yield point of the material.

2. Submarine bulkheads may be divided into the following types according to their design.

(a) Flat bulkheads. The methods of strength analysis of such bulkheads does not differ essentially from the one used in the analysis of ordinary emergency bulkheads of surface vessels. The most advantageous system of assembly of flat bulkheads is determined by local installation conditions. The foregoing depends chiefly on the design of their supporting contour and on the possibility of using the connections of the hull adjoining them for strengthening of the bulkhead.*

When a light platform is used as a supporting structure for the perpendicular girders of the bulkhead assembly, appropriate local rigidity of such a platform must be guaranteed, to avoid buckling at the floor of the platform in the region where the bulkhead abuts. To strengthen flat end bulkheads, they may be jointed to the torpedo tubes. For this purpose, the torpedo tubes must be securely attached to those parts of the light hull of the submarine located ahead of these bulkheads.

(b) Cylindrical bulkheads. These bulkheads are formed by the longitudinal walls of the wing or bottom tanks of the hull, and are designed with a cylindrical curvature. The transverse structure of such bulkheads appears as a portion of the composite frames of the hull. The strength of the plating of cylindrical bulkheads must be analyzed relative to stresses and also with respect to stability as a cylindrical shell with corresponding radius of curvature; thereby, the conclusions drawn from investigation of the strength of the plating of the solid hull must be taken into account.

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*Editor's Note: The author probably has in mind stiffening the bulkhead by securing it to hull flanges, etc.

(c) Spherical bulkheads.* The circular shape of the supporting contour of transverse bulkheads installed in submarines permits designing them spherically.

In the following, a practical method of analysis of spherical bulkheads is outlined.

18. ADVANTAGES AND DISADVANTAGES OF SPHERICAL BULKHEADS.

Due to the peculiar (circular) shape of the supporting contour of the pressure hull bulkheads of submarines, and also, owing to the comparatively large pressures which they must withstand, it appears advantageous to design them in spherical shape. As a result, they develop fundamentally only stresses which occur in a flexible membrane. Let us examine the direct advantages and disadvantages of such bulkheads, to determine the degree of expediency of their use -- depending on the conditions in each particular case.

1. From the point of view of economy of weight, a spherical bulkhead is notably more advantageous than a flat bulkhead. This appears to follow from the fact that in the connections of a flat transverse bulkhead, which are under bending, the stresses are distributed non-uniformly and consequently the material of these connections is exploited less effectively than in a spherical bulkhead, where the stresses over its entire area and through its thickness are practically the same.

From the structural and design point of view, spherical or dished bulkheads are advantageous because of the absence of stiffeners. The absence of such ribs eliminates significant construction and fabrication difficulties connected with their installation and facilitates making the bulkheads water-tight, especially in regions adjacent to the hull plating. However, production advantages of spherical bulkheads can be achieved only by proven mass-production methods, which require special and cumbersome equipment.

The presence of the platform of side or bottom tanks intersecting the bulkhead or abutting on it in the region of installation of the bulkhead, necessitates changing its true circular supporting shape. This, in turn, has an adverse effect on the performance of spherical bulkheads of the pure design. In such cases, the advantages of spherical bulkheads cannot be fully utilized; therefore, the degree of expediency of the application of spherical bulkheads as compared to flat ones depends on the conditions prevailing for each particular case.

2. As an inherent defect of spherical bulkheads may be mentioned the

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*Editor's Note: By spherical bulkheads, the Russians apparently mean "dished" structures.

very unequal resistance to water pressures on the opposite sides of the bulkhead. The resistance of a spherical bulkhead to the pressure of water on its convex side is greatly reduced since it is limited by the limit load, corresponding to the moment of loss of stability of the bulkhead, i.e. to the inception of bending in the opposite direction. This defect has no substantial significance only for such bulkheads designed to resist the water pressure only on one side and thus guarantee the safety of the vessel, such as the bulkheads which seal off the central control station compartment. Use of pure spherical bulkheads for other submarine hull compartments is somewhat detrimental to the safety of the vessel; however, it is generally practiced, but is not to be recommended. In such cases, spherical bulkheads must be strengthened additionally by a system of light stiffeners to increase the rigidity; these stiffeners are to be mounted on the concave side. Thus, with only a small increase in the weight of the bulkhead structure, a significant increase in its stability is obtained, i.e. of its resistance to the water pressure on its convex side.

19. PERFORMANCE CHARACTERISTICS OF A SPHERICAL BULKHEAD.

1. From the point of view of structural mechanics, a spherical bulkhead represents a thin shell, having the shape of a spherical segment, whose supporting edge is connected with a cylindrical shell which represents the hull plating of the submarine. The external load on this system consists of the water pressure acting on both of the aforementioned shells. Figure 47, a and b, shows the water pressure distribution during testing of the bulkhead strength (on the ship), while Figure 47, c and d, give the pressure distribution on the bulkhead for emergency operating conditions. It is easy to see that pressures for the cases shown in Figure 47, b and c, are less favorable than those shown in Figure 47, a and d. Similarly, it is evident that the difference between the water pressure on the convex and the concave surfaces of the bulkhead consists only in the reversal of the direction of the load acting; therefore, in both the foregoing instances of hydraulic loading, all the internal forces and moments, stresses and displacements will be identical in magnitude but opposite in sign.

The difference between the response of the bulkhead under a hydraulic pressure on the concave or convex side will consist in the fact that in the second case its useful performance may be disrupted as the result of loss of stability at a water pressure far below that which the bulkhead can withstand in the first case. Hence, it follows that the strength of a spherical bulkhead must be calculated with respect to stresses, for the case, of pressure acting on the concave side, under damage conditions (Figure 47 c), and for stability under the conditions shown for the dockyard tests (Figure 47 b).

2. The performance characteristics of a spherical bulkhead depend on the design of its connection to the hull plating, that is on the conditions

relative to the fastening of its supporting edge, as well as relative to the rotation angles and translational displacements in the plane of the supporting rim or edge.

Of the three reactions of the supporting edge shown in Figure 48, reactive moment $\mathfrak{M}$, transverse reaction H and longitudinal reaction T, only the latter appears in statically determinable and equal to

$$T = \frac{q\pi r^2}{2\pi r} = \frac{1}{2} q r$$

The magnitude of the reactive moment and of the transverse reaction depends on the degree of rigidity of the structure at the point where the spherical bulkhead abuts on the hull plating. The magnitude of these reactions may vary within limits ranging from zero to a certain maximum value corresponding to an absolutely rigid connection of the supporting edge, at which corresponding displacements of the supporting edge are absent.

The reactive (supporting) moment $\mathfrak{M}$ produces bending of the panel of the bulkhead in the region adjacent to the supporting rim. As theoretical investigations have shown, this bending is of a wave-like character and very quickly damps out as the distance from the supporting edge increases. Taking this into account, the stresses corresponding to the bending in bulkhead panels may be considered as purely local stresses, which do not affect the strength of the bulkhead, subjected only to loads of purely sporadic type. This circumstance permits us to simplify significantly the strength analysis of a bulkhead, by assuming in all cases that its edges are simply supported, despite the fact that in reality they will be clamped to a greater or lesser extent.

The transverse reactive stresses H, acting along the circumference of the supporting bulkhead, reveal a more substantial influence on its performance characteristics. For a visual presentation of this effect, we shall first examine the performance of the bulkhead in the absence of the transverse reaction H along its supporting contour, i.e. assuming that its supporting contour is free to move in its plane. In Figure 48, the broken line depicts the unloaded arc section of the bulkhead and the solid line shows the same section under loading by a uniformly distributed internal pressure. Under the action of the load, the plating of the bulkhead bulges, whereupon in the middle region of the bulkhead, tensile stresses develop in the plating, which, acting together with the longitudinal reactions T, compress the region of the bulkhead adjacent to its supporting contour. As a result, the radius of the supporting contour r diminishes and in the plating adjacent to the supporting contour compressive stresses arise equal to $E\Delta r/r$, where Δr is the decrease of the radius of the supporting contour. The compressive stresses may attain a magnitude at which the plating loses its stability and begins to buckle in the region of the supporting contour;

thereafter, this region ceases to perform its duty of a thrust member or "tie-bar" for the middle region of the bulkhead.

The longitudinal reaction T, in addition to its contribution to the compression of the supporting contour of the bulkhead mentioned above, also produces bending of the bulkhead which attains its greatest value close to the supporting edge and which quickly fades out as the distance from the edge increases.

If the supporting edge is strengthened by a reinforcing ring, or if it is fastened to such a ring (Figure 49), its displacement in the plane of the supporting contour will be made more difficult. As a result, the displacement Δr, and consequently the stresses corresponding to it, diminish accordingly due to the occurrence in this case of transverse reactions which elongate the bulkhead in the plane of its supporting contour. Thus, the supporting ring serves as a thrust member for the bulkhead which increases its inherent thrust resistance* and, by producing transverse tensile reactions, decreases the compressive forces and stresses in the vicinity of the supporting contour of the bulkhead while not causing a noticeable effect on the magnitude of the tensile forces and stresses in its central region.

The greater the area of the section of the supporting ring, or stated otherwise the greater the thrust resistance of the spherical bulkhead, the closer will such a bulkhead approximate a closed spherical shell subject to internal pressure.

However, even in the limiting case, in complete absence of displacements of the supporting edge of the bulkhead, full similarity between the performance characteristics of spherical bulkheads and spherical shells will not be attained. In the limiting case, the stresses in the meridional sections of the supporting edge of the bulkhead will be absent, while, under the operating conditions of a spherical shell, they should be those as in any section of the central region of the bulkhead. Stresses in the conical sections of spherical bulkheads close to the supporting contour, which are always tensile stresses, increase with the increase of the thrust of the bulkhead. In the limiting case, they approximate closely the stresses in the conical sections of the central region of spherical bulkheads, i.e., they approximate the stresses corresponding to the operating conditions of a closed spherical shell. On the basis of what has been said, it is possible to conclude that the expediency of increasing the area of the section of the supporting ring (thrust) is subject to a certain limit determined by the identity of the material properties composing its supporting edge in conical and in meridional sections.

*Editor's Note: The Russian here uses the term "self-thrust" or, "automatic thrust", which appears to be merely an augmentation of the natural thrust before addition of this strengthening ring.

20. STRESS ANALYSIS OF SPHERICAL BULKHEADS

1. The analysis of spherical bulkheads with respect to stresses is reduced to the investigation of the deformation of a thin shell, having the shape of a spherical segment and loaded by a given internal pressure q and unknown transverse reactions H, distributed uniformly along the periphery of the supporting contour of the shell. The magnitude of the reactions must be determined from the condition that the displacements of the supporting contour of the shell must equal the displacements of the supporting ring under the same but oppositely directed reactions (Figure 48).

If the spherical segment being examined is part of a closed spherical shell, the transverse reactive forces on the supporting contour H_0, the displacements of the supporting contour Δr and the stresses in any section of the shell σ, would be determined, respectively, by the following expressions (Figure 50):

$$H_0 = \frac{qR}{2} \cos \phi_0$$

$$\Delta r = \frac{qR}{2tE} r; \qquad \sigma = \frac{q\pi R^2}{2\pi Rt} = \frac{qR}{2t};$$

wherein R is the radius of the spherical shell; ϕ_0 is half the central angle limiting the spherical segment; r is the radius of the supporting contour; t is the thickness of the shell.

If, on the deformation of the spherical segment, we superimpose the deformation found in the segment due to the application of the transverse forces alone, equal to $H_0 - H$ and applied to the supporting edge as shown in Figure 51 a, then, as a result, the deformation of the spherical segment under the action of a load corresponding to Figure 51 b which is of interest here, will be obtained.

Equating the displacement of the supporting contour for this deformation to the displacement of the supporting ring, i.e. to the quantity,

$$\Delta r = \frac{Hr}{EF} r = \frac{HR^2}{EF} \sin^2 \phi_0$$

where F is the area of the section of the ring, we shall get a relation from which the unknown transverse reaction H can be found.

Thus, the solution to the problem being considered is reduced to the investigation of the deformation of a spherical segment, loaded only by transverse forces, uniformly distributed along its edge. The solution to this problem can be obtained easily by using the appropriate results of the approximate theory of the deformation of a spherical shell. Given below are

the expressions, obtained by the method indicated, for computation of the design stresses in a spherical bulkhead, assuming its edge to be freely supported.*

2. As design stresses in a spherical bulkhead, we must take the largest uniformly distributed stresses in meridional and conic sections as determined by the following expressions:

stresses in the meridional and conic sections outside the region of the supporting contour:

$$\sigma_1 = \frac{q\pi R^2}{2\pi R t} = \frac{qR}{2t} \qquad [1]$$

stresses in the conic sections within the region of the supporting contour:

$$\sigma_2 = \frac{qR}{2t}\left[1 - \frac{\frac{rt}{F}\cos\phi_0 + 1 - \mu}{\frac{rt}{F} + A}\cos\phi_0\right] \qquad [2]$$

stresses in the meridional sections within the region of the supporting contour:

$$\sigma_3 = \frac{qR}{2t}\left[1 - \frac{\frac{rt}{F}\cos\phi_0 + 1 - \mu}{\frac{rt}{F} + A} A\right] \qquad [3]$$

reduced working stress outside the region of the supporting contour:

$$\sigma_{1np} = \sigma_1(1 - \mu) = 0.7\,\sigma_1 \qquad [4]$$

reduced stress in the conic sections within the region of the supporting contour:

$$\sigma_{2np} = \sigma_2 - \mu\sigma_3 \qquad [5]$$

reduced stress in the meridional sections of the supporting contour:

$$\sigma_{3np} = \sigma_3 - \mu\sigma_2 \qquad [6]$$

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*Markova, L. G.: "Method of Strength Analysis of Spherical Bulkheads of Submarines", Collection, NIVK, Number 6, 1936.

Stresses in the sections of the thrust ring must evidently be equal to the reduced stresses in the meridional sections of the supporting edge σ_{3np}.

In these expressions, (Figure 49), R is the radius of the spherical bulkhead; r is the radius of the supporting edge of the bulkhead; t is the thickness of the bulkhead; q is the design pressure on the bulkhead; μ is Poisson's Ratio ($\mu \approx 0.3$); F is the area of the section of the thrust ring; $A = 2.58\sqrt{r^2/tR}$ is a conventional notation.

In calculation of the area of the section of the supporting ring F, there must be included, in addition to the area of the connections especially designed to produce the thrust resistance of a spherical bulkhead, also the area of that portion of the hull plating immediately adjacent to these connections.

The magnitude of the longitudinal and transverse reactions of the supporting contour, with respect to which the strength of the welded or riveted connections of the spherical bulkhead to the hull plating and to the thrust ring must be checked, are determined by the following expressions:

longitudinal reactions:

$$T = \frac{q\pi r^2}{2\pi r} = \frac{qr}{2} \qquad [7]$$

transverse reactions (thrust):

$$H = \frac{F\sigma_{3np}}{r} \qquad [8]$$

21. EFFECT OF THE CROSS-SECTIONAL AREA OF THE THRUST RING.

1. The magnitude of the design stresses, which determine the strength of a spherical bulkhead, depends strongly on the magnitude of the cross-sectional area of the thrust ring; this dependency increases as the central angle of the spherical bulkhead ϕ_0 decreases.

Below are given numerical examples, illustrating the influence of the area of the section of the thrust ring, on the stresses of a spherical bulkhead having a central angle usually found in practice in accordance to requirements for the mounting of spherical bulkheads in submarines.

The relative magnitude of the area of the section of the thrust ring may be expressed as the thrust coefficient χ, varying in the range from zero (for F = 0) to unity (for F = ∞) and is determined by the following expression:

$$\chi = \frac{F}{F + tr} \qquad [9]$$

The dependence of design stresses in the conic (σ_{2np}) and in the meridional (σ_{3np}) sections of a spherical bulkhead, on the value of the thrust coefficient χ is shown in Figure 52, applicable to the example cited below. From an examination of the curves in Figure 52, it is apparent that the effect of the area of the section of the thrust ring increases inversely with the thrust coefficient.

On the basis of the foregoing and considering structural and production difficulties in connection with increasing the area of the section of the thrust ring, the conclusion must be made concerning the ineffectiveness of increasing of the section of the thrust ring in excess of a certain limit, beyond which the advantage of such rings becomes comparatively small.

Using the coefficient of thrust ($\chi<0.5$), which can be attained rather easily in practice, the largest reduced stresses in spherical bulkheads are found to act in its meridional sections, in the region of the supporting edge (σ_{3np}); these stresses are compressive and equal to the compressive stresses in sections of the thrust ring.

Assuming a reasonable value for the allowable compressive stress for the bulkhead material, and by using Figure 52, the minimum required area of the section of the thrust ring may be found, guaranteeing the strength of the bulkhead.

Example:

To compute the design stresses in a spherical bulkhead, using the following data:

radius of the bulkhead	R = 850	[cm]
radius of the supporting contour	r = 230	[cm]
thickness of the bulkhead	t = 2.5	[cm]
thickness of the hull plating	t = 1.3	[cm]
pressure on the bulkhead	q = 6	[atm]

The area of the section of the thrust ring (Figure 53):

$$F = 3(20 \times 2.5) + 20 \times 1.3 = 176 \ [\mathrm{cm}^2]$$

$$\sin \phi_0 = \frac{r}{R} = \frac{230}{850} = 0.27 \qquad \cos \phi_0 = 0.963$$

$$\frac{rt}{F} = \frac{(230 \times 2.5)}{176} = 3.25$$

$$A = 2.38\sqrt{\frac{230^2}{2.5 \times 850}} = 11.9$$

$$\chi = \frac{F}{F + rt} = \frac{176}{176 + 230 \times 2.5} = 0.234$$

The stresses in the meridional and conic sections, outside of the supporting contour, by Equation [1] are:

$$\sigma_1 = \frac{qR}{2t} = \frac{(6 \times 850)}{(2 \times 2.5)} = 1020 \quad \text{atm}$$

The stresses in the conic sections in the region of the supporting contour by Equation [2] are:

$$\sigma_2 = 1020 \left[1 - \frac{3.25 \times 0.96 + 0.7}{3.25 + 11.9} 0.963 \right] = 1020 \cdot (1 - 0.242) = 775 \quad \text{atm}$$

Stresses in the meridional sections in the region of the supporting contour by Equation [3] are:

$$\sigma_3 = 1020 \left[1 - \frac{3.25 \times 0.96 + 0.7}{3.25 + 11.9} 11.9 \right] = 1020 (1 - 3.00) = -2040 \quad \text{atm}$$

Reduced stresses by Equation [4], [5] and [6] are:

$$\sigma_{1np} = 0.7\,\sigma_1 = 0.7 \times 1020 = 714 \quad \text{atm}$$

$$\sigma_{2np} = \sigma_2 - \mu\sigma_3 = 775 + 0.3 \times 2040 = 1390 \quad \text{atm}$$

$$\sigma_{3np} = \sigma_3 - \mu\sigma_2 = 2040 - 0.3 \times 775 = 2260 \quad \text{atm}$$

Stress values, in the absence of a thrust ring ($F = 0$, $X = 0$), are:

$$\sigma_1 = 1020 \quad \text{atm}$$

$$\sigma_2 = 1020 (1 - \cos^2 \phi_0) = 1020 (1 - 0.927) = 75 \quad \text{atm}$$

$$\sigma_3 = 1020 (1 - \cos \phi_0 A) = 1020 (1 - 0.963 \times 11.9) = -10700 \quad \text{atm}$$

$$\sigma_{1np} = 714 \quad \text{atm}$$

$$\sigma_{2np} = 75 + 0.3 \times 10700 = 3280 \quad \text{atm}$$

$$\sigma_{3np} = -10700 - 0.3 \times 75 = -10720 \quad \text{atm}$$

Stress values, for complete clamping of the supporting contour, are:

$$F = \infty \qquad X = 1$$

$$\sigma_1 = 1020 \ [\text{atm}]$$

$$\sigma_2 = 1020\left(1 - \frac{0.7}{11.9}\,0.963\right) = 962 \ [\text{atm}]$$

$$\sigma_3 = 1020\left(1 - \frac{0.7}{11.9}\,11.9\right) = 306 \ [\text{atm}]$$

$$\sigma_{1np} = 714 \ [\text{atm}]$$

$$\sigma_{2np} = 962 - 0.3 \times 306 = 870 \ [\text{atm}]$$

$$\sigma_{3np} = 306 - 0.3 \times 962 = 0$$

The dependence found between the reduced stresses and the value of the thrust coefficient is shown in Figure 52. When this dependency is known, the area of the section of the thrust ring required for strength and solidity of the bulkhead can be determined more exactly. Thereupon, the forces may be found for which the strength of the welded or riveted connections of the spherical bulkhead to its thrust ring and to the hull plating must be assured.

Longitudinal reactions, by Equation [7]

$$T = \frac{qr}{2} = \frac{(6 \times 230)}{2} = 690 \ [\text{kg cm}^{-1}]$$

Transverse reactions, (thrust), by Equation [8]

$$H = \sigma_{3np}\frac{F}{r} = \frac{(2260 \times 176)}{230} = 1730 \ [\text{kg cm}^{-1}]$$

22. STABILITY ANALYSIS OF SPHERICAL BULKHEADS

1. **When the external load is applied from the convex side of a spherical bulkhead, the analysis with respect to stability of its shape should be carried out, because in this case the stability of a spherical bulkhead is lost at a loading, which produces in its sections relatively small stresses. For such a calculation the following formula is used, developed by Zoelly, for a complete spherical shell**:

$$q_{Kp} = \frac{2E}{\sqrt{3(1-\mu^2)}}\left(\frac{t}{R}\right)^2 = 240\left(\frac{100t}{R}\right)^2 \qquad [10]$$

where q_{Kp} is the critical pressure [atm]; t is the thickness of the shell [cm]; R is the radius of the shell [cm]; E is Young's Modulus, usually taken as 2×10^6 [kg/cm^2]; μ = Poisson's Ratio = 0.3.

However, as tests show, Zoelly's formula turns out to be very inexact, giving much too large values for the critical load both for a complete spherical surface, as also to a larger degree for the portion of the spherical surface, bounded by some supporting contour. (This latter is the case

of a spherical bulkhead).

By using certain available experimental data, Zoelly's formula can still be used, by substituting into it appropriate correction factors, which consider not only the inexactitude of the formula itself, but also the effect of the shape and degree of rigidity of the supporting contour of the spherical shell.

As a result of the evaluation of available experimental data, the following formula can be proposed for computation of the value of the indicated correction coefficients, (Figure 54).*

$$k = 0.45\,[\sin\phi_0 + (1 - \sin\phi_0)\chi] \qquad [11]$$

where ϕ_0 is half the central angle, bounding the spherical shell; $\chi = F/F + tr$ = the coefficient of rigidity of the supporting contour of the shell; F is the area of the thrust (supporting) ring; r is the radius of the supporting contour.

Example:

To determine the critical loading on a spherical bulkhead of thickness t = 25 mm, with radius R = 8.5 m; supporting contour radius r = 2.3 m with section shown in Figure 53 (F = 176 cm^2).

Critical loading according to Zoelly's Formula [10],

$$q_{Kp} = 240\left(\frac{100 \times 2.5}{850}\right) = 20.8\ [\mathrm{atm}]$$

Coefficient of rigidity of supporting contour:

$$\chi = \frac{F}{F + tr} = \frac{176}{176 + 2.5 \times 230} = 0.234$$

Half of central angle:

$$\sin\phi_0 = \frac{230}{850} = 0.27$$

Correction factor by Equation [11]:

$$k = 0.45\,[0.27 + (1 - 0.27)\,0.234] = 0.20$$

Critical load:

$$q_{Kp} = 0.20 \times 20.8 = 4.2\ [\mathrm{atm}]$$

*For an angle ϕ_0 of the order of 15°.

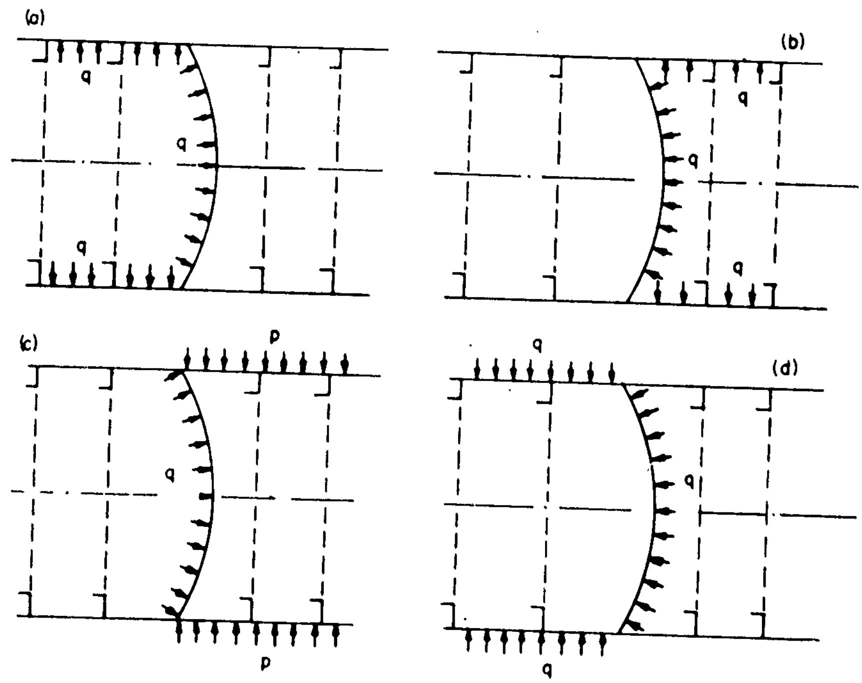


Figure 47

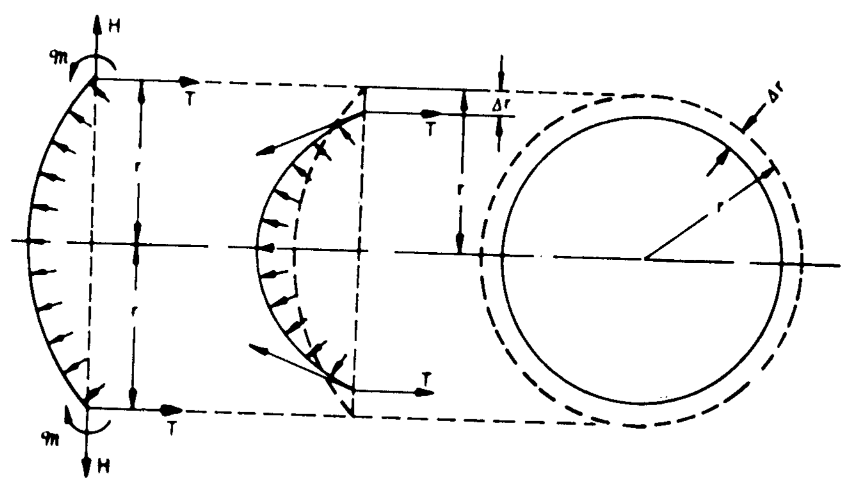


Figure 48

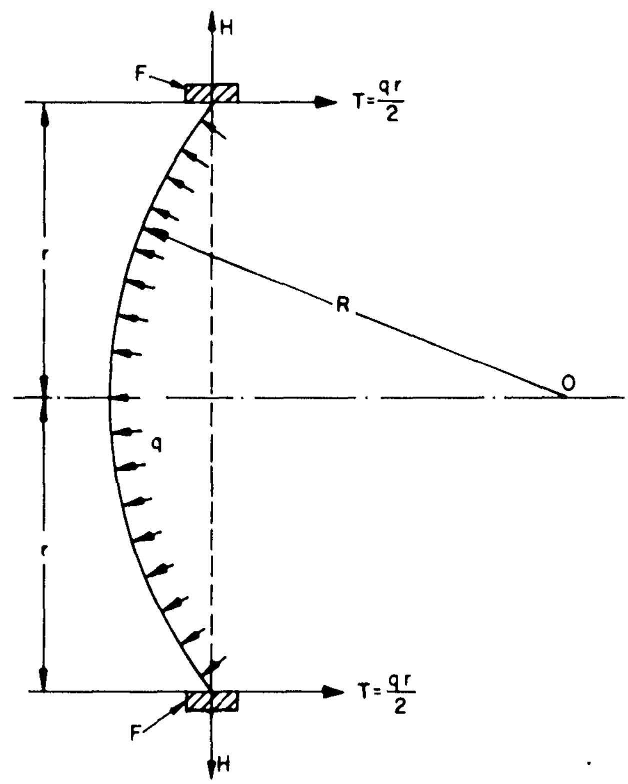


Figure 49

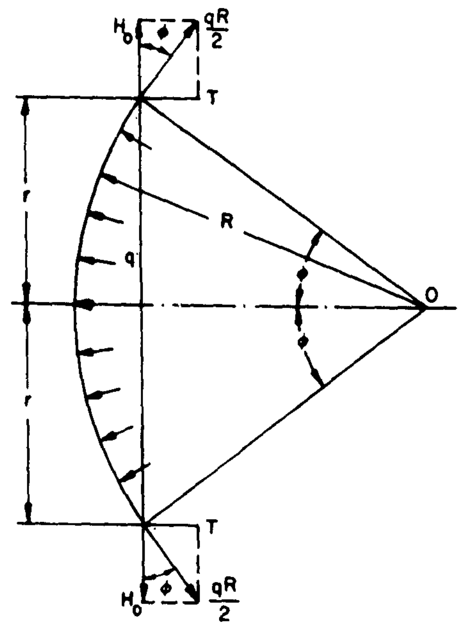


Figure 50

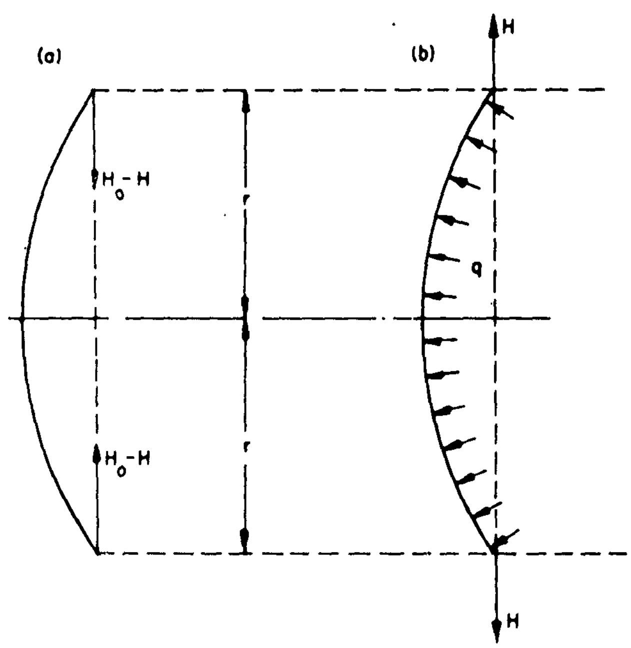
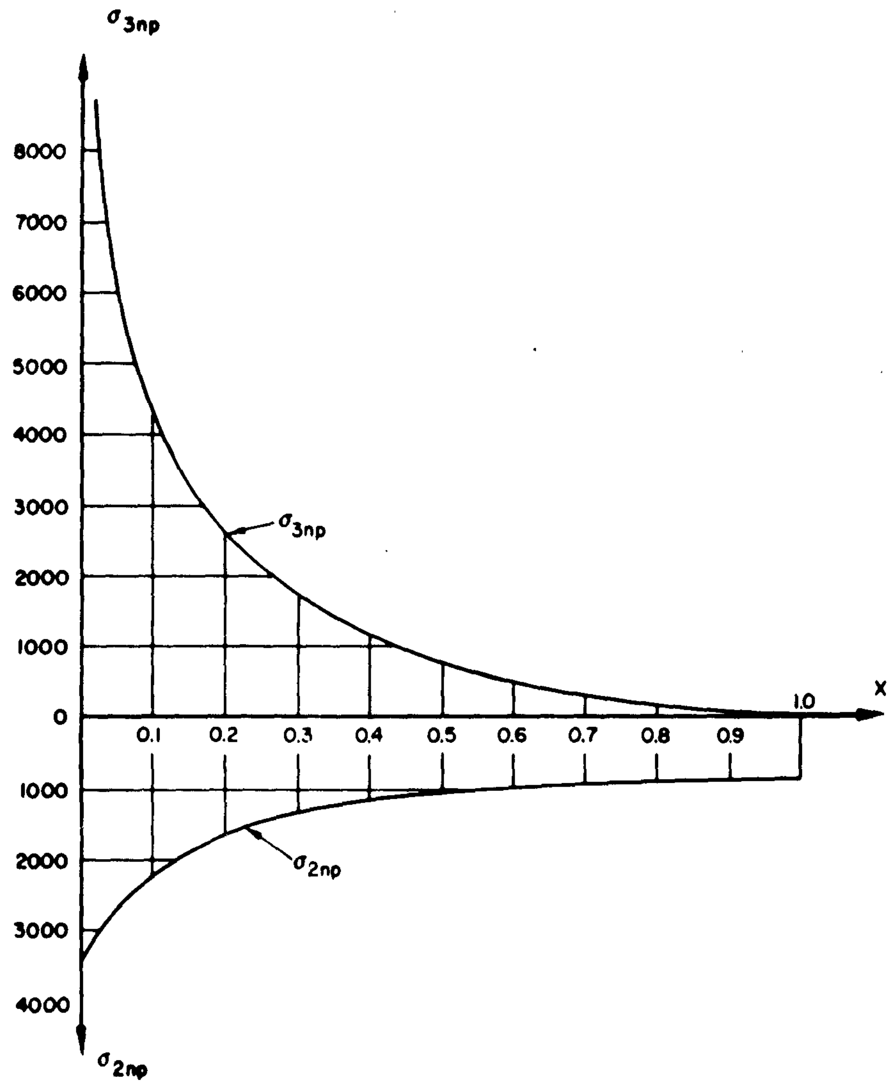


Figure 51



σ_{3np}
8000
7000
6000
5000
4000
3000
2000
1000
0
1000
2000
3000
4000
σ_{2np}
σ_{3np}
σ_{2np}
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
X

Figure 52

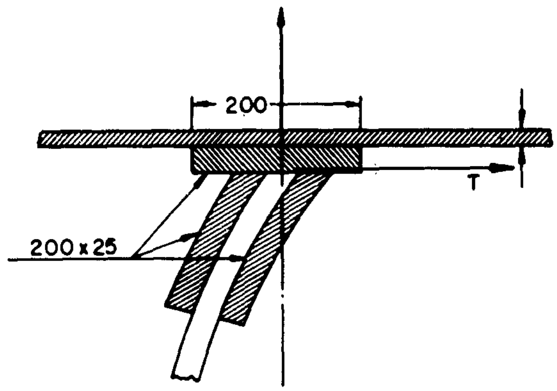


Figure 53

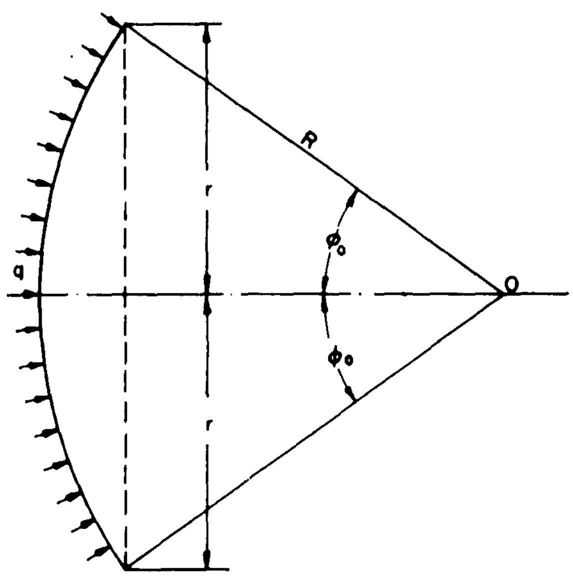


Figure 54

CHAPTER VI

REINFORCEMENT OF THE HULL

In designing submarine hulls, as well as hulls of surface vessels, we must deal with the problem of strength analysis and structural design of different types of local reinforcements of the joints (seams, connections) of the basic hull. Local reinforcements of the hull are intended to respond to the various local loads acting on the hull, or to restore the strength of connections in such points where they have been weakened for one reason or another.

Calculation of the strength and the construction of submarine hull reinforcements, designed to assume various local loads acting on the hull, include for example reinforcements for ice loading, reinforcements under installations and mechanisms mounted in the ship, reinforcements for hatch openings and hawse pipes, and is in no way fundamentally different from calculation of such reinforcements for surface ship hulls. Specifically peculiar reinforcements for submarine hulls, which therefore require special investigation, are only reinforcements designed for the restoration of the strength of the hull at points where it is weakened. Under the latter category we may enumerate points of deviation from the true circular shape of the frames and plating of the pressure hull and various openings in it.

In addition to these basic types of reinforcements, must be noted the necessity in certain cases of employing local reinforcements to submarine hulls only for the duration of tests by internal water pressure while in the dockyard. The strength of a submarine hull, as true of any other engineering structure, is determined by the strength of such weakened, even though isolated, points of the hull, which result from possible neglect or omission during design and construction. Therefore, no less attention must be paid to the problem of determining the appropriate location for reinforcement of wittingly weakened portions of the hull than is given to the question of the strength and stability of the entire hull.

In the present chapter are given the general theoretical fundamentals, structural requirements and practical standards relating to the above indicated types of reinforcement of submarine hulls.

23. THEORETICAL FUNDAMENTALS FOR REINFORCEMENTS OF THE HULL IN ZONES DEVIATING FROM TRUE CIRCULAR SHAPE.

1. In solving the problems related to assuring the strength of submarine hulls, we must deal in practice with instances of significant departure from the true circular shape of the frames and plating of the pressure hull. Incorrect bending of frames and plating occur either during construction of the hull or during operation of the submarine. In the present case, we are concerned with incorrect bending which exceeds the limits

and specifications established in industrial standards, in conjunction with the usual and conventional methods applied in calculation of hull stability. Decrease of hull strength, unavoidable under excessive and incorrect bending, must be compensated by adequate reinforcement of the hull or by decreasing the limiting depth of submersion.

A detailed analytical investigation of the effect of initial bending in a ring and a cylindrical shell on their strength is given in Chapter V of Part 2. The results of this investigation may be applied completely and used for a quantitative evaluation of the unfavorable effect on the stability of the hull of an initial, incorrect bending of its frames and shell (plating).

2. Let us examine first the phenomenon of longitudinal compression of a bar, having an initial bending of the nature and form of a curved line, expressed by the equation $y = f(x)$.

As is known, every such curve can be expressed as the sum of orthogonal curves, formed according to modes of the free oscillations of the bar, i.e. in the given case in the form of sinusoids with a different number of half-waves (Figure 55). Such a presentation is equivalent to expansion of the $y = f(x)$ into a Fourier series, i.e.

$$y = f(x) = a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{2\pi x}{l} + a_3 \sin\frac{3\pi x}{l} + \ldots + a_n \sin\frac{n\pi x}{l},$$

where a_1, a_2, a_3 ... a_n are coefficients of the series representing the maximum bending deflections of the component sinusoids.

To determine the magnitude of these coefficients, for example of coefficient a_m, the series must be multiplied by $\sin m\pi x/l$ and integrated within the limits from 0 to l.

Noting that $\int_0^l \sin^2(m\pi x/l)\, dx = 1/2\, l$ and all the remaining integrals with the product of the sines are equal to 0, we get:

$$a_m = \frac{2}{l}\int_0^l f(x) \sin\frac{m\pi x}{l}\, dx \qquad [1]$$

The value of the integral on the right-hand side of Equation [1], can be readily computed by the tabular method according to the known values of the function $f(x)$, i.e. according to the known values of the ordinates of the bent axis of the bar.

With longitudinal compression of the curved (bent) bar, there occur in its sections, in addition to compressive forces, bending moments which produce a bending of the bar and which are equal to the product of the compressive force and the magnitude of the total, that is the initial

and the elastic, deflections of the bar.

The bending of the bar may be expressed as the sum of the bendings of the bars, initially bent according to the types of sinusoids indicated above, with a different number of half-waves.

Let us examine one of such partial bendings, for example with m half-waves, determined by the initial shape $y_m = a_m \sin m\pi x/l$. Designating the largest elastic deflection for it by Δa_m, i.e. having taken for it the elastic curve as $\Delta y_m = \Delta a_m \sin m\pi x/l$, we get the following differential equation for the bending:

$$M_x = P(a_m + \Delta a_m)\sin\frac{m\pi x}{l} = -EI(\Delta y_m)'' = EI\frac{m^2\pi^2}{l^2}\Delta a_m \sin\frac{m\pi x}{l}$$

Whence

$$\Delta a_m = a_m\frac{1}{\frac{EI\pi^2 m^2}{Pl^2} - 1} = a_m\frac{1}{\frac{P_{эm}}{P} - 1} \qquad [2]$$

$$a_m + \Delta a_m = a_m\frac{1}{1 - \frac{P}{P_{эm}}}, \qquad [3]$$

where*

$$P_{эm} = \frac{EI\pi^2}{l^2}m^2 \qquad [4]$$

The total bending moment in sections of the bar is:

$$\sum M_x = P\sum(a_m + \Delta a_m)\sin\frac{m\pi x}{l} = P\sum\frac{a_m}{1 - \frac{P}{P_{эm}}}\sin\frac{m\pi x}{l} \qquad [5]$$

---

*Editor's Note: The Russian word for <u>Euler</u> starts with the character э.

The quantity $P_{эm}$, determined by Equation [4] represents the Euler loading of the bar, which corresponds to the bending at the loss of stability with m half-waves; this follows from the fact that with $P = P_{эm}$, the deflection of the bar Δa_m becomes infinitely large [Equation (2)].

The minimum value of Euler's loading is found to be for $m = 1$, that is for a bending of the bar in one half-wave. This form of initial bending of the bar is the least favorable, since for a given value of the compressive load P, it affects, more than all other forms, the magnitude of the bending moments in the sections of the bar. This follows directly from Equation [5].

In Equation [5] for the bending moments, the first term of the series is predominant, corresponding to the value $m = 1$, especially under a compressive load P acting on the bar which approximates closely Euler's loading $P_{э1} = EI\pi^2/l^2$. Therefore, we may confine ourselves practically, in the investigation of interest here, to consider in this expression only the first term, i.e. to consider the greatest bending moment acting in the central section of the bar (with $x = 1/2l$) equal to:

$$M_{max} = P\frac{a_1}{1-\frac{P}{P_{э1}}} = P\frac{a_1}{1-\frac{\sigma_0}{\sigma_{э1}}}$$

where a_1 is the initial deflection of the bar determined by Equation [1] with $m = 1$; $\sigma_0 = P/F$ is the compressive stress in the sections of the bar corresponding to the compressive load P, where F is the area of a section of the bar; $\sigma_{э1} = P_{э1}/F$ is Euler's stress of the bar.

The maximum stress in the central section of the bar (from compression and from bending) is

$$\sigma_{max} = \sigma_0 + \frac{M_{max}}{W} = \sigma_0 + \frac{M_{max}}{\eta hF} = \sigma_0\left[1 + \frac{1}{\eta h}\cdot\frac{a_1}{1-\frac{\sigma_0}{\sigma_{э1}}}\right] \qquad [6]$$

where $W = \eta hF$ is the moment of (resistance) of a section of the bar, equal to the product of the area of the section F by the height of the section h and by the coefficient of efficiency η which depends on the shape of the section.

Using the expression obtained, Equation [6], the largest initial bending deflection of the bar can be determined for which the largest stress in its sections will not exceed a given limit. Indicating this limiting (critical) stress by σ_K we find the following expression for the admissible initial bending to be:

$$\frac{a_1}{h} \leq \eta\left(\frac{\sigma_K}{\sigma_0} - 1\right)\left(1 - \frac{\sigma_0}{\sigma_{э1}}\right) \qquad [7]$$

3. On the basis of the investigation outlined above, the following conclusions can be drawn with respect to the influence of initial deflection of the bar on compressive deformation.

In compressing a bar having an initial deflection, bending stresses develop in its sections whose magnitude depends not only on the magnitude of the compression loading and on the magnitude of the deflection, but also on the form of this bending. As the compression loading P increases, the effect of the initial deflection increases sharply, which has the form of one half-wave, i.e. a form corresponding to that characteristic of loss of stability of the bar. Under a compression loading close to Euler's loading of a bar, such a deflection shows a decisive influence on the deformation of the compressed bar, and therefore, it is possible to confine calculation to the effect of only this form of the initial bending of the bar. In this case, the magnitude of the admissible initial deflection can be found by Equation [7] as a function of the design compressive stress in the bar σ_0, of Euler's stress $\sigma_{э1}$, of the value of the limiting stress of the material σ_K, and of the shape of the section, determined by the coefficient of efficiency, with respect to the compressed edge of the section, η.

The presence of initial bending in the bar may very much diminish its resistance to the action of a compressive load, not only because the largest total stresses then occurring in the sections of the bar may be dangerous to the structural material, but chiefly, because these stresses diminish Euler's loading on the bar as a result of decrease of Young's Modulus E entering into the theoretical formula for the deformation of this loading (Part 2, Section 33). The decrease of Euler's loading, in turn, produces a further increase of maximum stresses in the cross-sections of the bar, as may be seen from Expression [6]. The progressive character of the unfavorable effect of the initial bending of a bar on its resistance to the effect of a compressive load may lead to a great decrease of its Euler loading, i.e. of its failure load, as compared with the magnitude of this loading when determined by the usual theoretical formula.

4. Any deviation from the true circular shape of the frame may be expressed in terms of a sum of individual deviations therefrom, having an exact sinusoidal form with a different number of waves, i.e. in the form of a series (Figure 56):

$$\omega_\alpha = \sum f_n \sin n\alpha \qquad [8]$$

where ω_α is the total magnitude of the deviation from a circle in the section of the frame, determined by angle α ; the angle α is measured from the section for which $\omega = 0$; f_n is the magnitude of the maximum deviation corresponding to the form of a sinusoid with n waves; $n \geq 2$ is an integer equal to the number of waves of which the sinusoids are composed, constituting the

deviations of the frame from a true circle.

If the function ω_α is known, then the coefficients f_n of the series [8], can be found by multiplying it by sin $n\alpha$ and subsequent integration within the limits from 0 to 2π:

$$f_n = \frac{1}{\pi}\int_0^{2\pi} \omega_\alpha \sin n\alpha d\alpha \qquad [9]$$

With the application on the frame of a uniformly distributed compressive loading p along its circumference, there occur in its sections in addition to compressive forces $S = pr$, also bending moments equal to the product of the compressive force S and the magnitude of the total, i.e. of the initial and elastic, deviations of the frame from true circular shape. This bending of the frame can be expressed as the sum of the bending of the rings, initially bent according to the true sinusoidal curves mentioned above.

Let us examine separately one such component bending, determined by the initial form $\omega_n = f_n \sin n\alpha$ and by its elastic form $\Delta\omega_n = \Delta f_n \sin n\alpha$.

By substituting the expressions for ω_n and $\Delta\omega_n$ into the well known differential equation for the bending of a ring, we get:

$$\frac{EI}{r^2}\left[\frac{\partial^2 \Delta\omega_n}{\partial\alpha^2} + \Delta\omega_n\right] = -M_\alpha = -S(\omega_n + \Delta\omega_n) = -pr(\omega_n + \Delta\omega_n)$$

$$\Delta f_n = f_n \frac{1}{\frac{EI}{pr^3}(n^2-1) - 1} = f_n \frac{1}{\frac{p_{\vartheta n}}{p} - 1} \qquad [10]$$

$$f_n + \Delta f_n = f_n \frac{1}{1 - \frac{p}{p_{\vartheta n}}} \qquad [11]$$

where

$$p_{\vartheta n} = \frac{EI}{r^3}(n^2 - 1) \qquad [12]$$

The total bending moment in the sections of the frame is:

$$\sum M_\alpha = S\sum(f_n + \Delta f_n)\sin n\alpha = pr\sum \frac{f_n}{1 - \frac{p}{p_{\text{э}n}}}\sin n\alpha \qquad [13]$$

Equation [12] determines the Euler loading of the frame corresponding to a bending with n waves. Since the magnitude of this loading is found to be least for $n = 2(p_{\text{э}2} = 3EI/r^3)$, i.e. for an elliptical bending of the frame, then such a type of initial bending of the frame appears the least favorable from the point of view of its effect on the magnitude of the bending moments in the frame sections, especially if the external loading p, is close to the Euler loading ($P_{\text{э}} = 3EI/r^3$) acting on the frame.

In this case, it is possible practically to limit ourselves by retaining only the first term in the general expression, Equation [13], for bending moments in the sections of the frame, i.e. to consider the maximum bending moment as equal to

$$M_{max} = pr\frac{f_2}{1 - \frac{p}{p_{\text{э}2}}} = pr\frac{f_2}{1 - \frac{\sigma_0}{\sigma_{\text{э}}}} \qquad [14]$$

where f_2 is the value of the greatest deviation of the frame from true circular shape, corresponding to an elliptical shape of the deviation (n = 2); $\sigma_0 = pr/F$ is the compressive stress in sections of the frame corresponding to the applied loading p where F is the area of the section of the frame; $\sigma_{\text{э}} = p_{\text{э}2}/F$ is Euler's stress of the frame.

The maximum total stress in the frame (due to compression and bending) is:

$$\sigma_{max} = \sigma_0 + \frac{M_{max}}{W} = \sigma_0\left[1 + \frac{1}{\eta h}\frac{f_2}{1 - \frac{\sigma_0}{\sigma_{\text{э}}}}\right] \qquad [15]$$

where W is the moment of resistance of the section of the frame with respect to its compressed edge; $W = \eta hF$ where h is the height of the section and η is the coefficient of efficiency.

The largest admissible deviation of the frame from a true circular shape, at which the maximum stress in it σ_{max} will not exceed the value of the specified limiting stress for the frame material σ_K, will be determined by the expression

$$\frac{f_2}{h} \leq \eta\left(\frac{\sigma_K}{\sigma_0} - 1\right)\left(1 - \frac{\sigma_0}{\sigma_э}\right) \qquad [16]$$

In the above investigation, the frame was considered as an isolated ring without taking into account the supporting effect on the deformation of frames of rigid transverse bulkheads of the hull (Part 2, Section 17). The favorable effect of transverse bulkheads on the deformation of bending and on the stability of the frames located in the span between bulkheads may be considered by an appropriate increase of the value of Euler's stress $\sigma_э$, entering the expressions for an isolated frame found above. The increased Euler's stress, corresponding to a pressure on the hull at which one of its elements between transverse bulkheads loses stability, may be found with the aid of the following expression:

$$\sigma_э = \frac{q_э l r}{F} \qquad [17]$$

where $q_э$ is the intensity of the pressure at which the portion or element of the hull between transverse bulkheads loses its stability [see Part 2, Section 18, Equation (52)]; l is the distance between frames; F is the sectional area of the frame (together with the adjacent strip or belt of plating).

5. Deviations of the hull plating from a true circular shape in the spans between frames usually occur as local smooth bulges or depressions (dents) between the frames, which extend (transversely) almost to the length of those half-waves arising in the plating when its stability fails under external water pressure. On the basis of what has been said in Paragraph 1, as well as in the preceding paragraph, it should be clear that precisely such a shape of initial undesired bending of the plating, coincident with the shape of its loss of stability, must be most unfavorable from the point of view of resistance to the action of compressive and bending loads. Under such conditions of least favorable type of initial bending of the plating, the maximum stress in its longitudinal section in the span between frames can be found by Equation [26] (Part 2, Section 31), assuming in this expression $n = n_1$, that is,

$$\sigma_{max} = \sigma_0\left[1 + \frac{6f_0}{h}\,\frac{1}{1 - \frac{\sigma_0}{\sigma_э}}\right] \qquad [18]$$

where h is the thickness of the plating; f_0 is the greatest initial bending deflection; σ_0 is the stress in the plating at the design pressure; $\sigma_э$ is Euler's stress in the plating.

The greatest admissible deviation of the plating from a true circular shape, at which the greatest stress in it does not exceed the specified limiting stress σ_K is determined by:

$$\frac{f_0}{h} \leq \frac{1}{6}\left(\frac{\sigma_K}{\sigma_Q} - 1\right)\left(1 - \frac{\sigma_0}{\sigma_э}\right) \qquad [19]$$

6. From comparison of the expressions found in the foregoing, which determine the effect of deviation from a true circular shape on the strength of frames and plating, with the analogous expressions found in Paragraph 2 of this section for a bar under compressions, it is evident that all these expressions are completely identical. Therefore, the general conclusions made in Paragraph 3 for the case of a compressed bar may be applied in full, both to the frames and to the plating of the circular pressure hulls of submarines. It is necessary only to point out that an incorrect initial bending of a frame (in the region of its concave portion) affects unfavorably not only the strength of the frames themselves, but the strength of the plating between the frames as well by increasing the compressive stress σ_0 acting in the plating and thus lowering its resistance as evident from Equations [18] and [19].

24. PRACTICAL STANDARDS AND REQUIREMENTS.

1. As a result of the unfavorable effect of an incorrect initial bending of frames and plating on their strength under compressive loading, in practice there is noted a discrepancy between full-scale tests on the prototype and model tests of these structures using design data from corresponding theoretical formulas.* To eliminate such discrepancies, practical correction coefficients are introduced into the theoretical formulas which are correlated with the data found from full-scale and model tests, made on structures containing deviations from true circular shape which are unavoidable due to the exigencies of production conditions (Part 2, Section 34).

If deviations from a true circular shape, in excess of standard production specifications and tolerances established, which can be traced to production defects, or which result from operation of the submarine, are detected in its hull, then all such hull areas must either be repaired or reinforced to eliminate possible weakening with respect to the assumptions made in design. The expressions obtained in the preceding paragraphs can be used both for establishment of the above mentioned standard production tolerances and also for the calculation of hull reinforcements in such zones

*Editor's Note: This discrepancy between full-scale and model tests is usually termed "scale-effect" in the U. S.

where the deviation of frames and plating from a true circle exceeds production allowances.

The value of production tolerances, established on the basis of relevant computations together with previous testing of submarine structures can be taken appropriately as follows: for the hull plating an order of 1% of the thickness of the plating, and for the frames -- an order of 0.25% of the frame radius. Such rigid standards have been specified on the assumption that the form of deviation of frames and plating from the correct circular shape coincides with their form of loss of stability. In the case of a more favorable shape of initial bending, the indicated standards may obviously be improved correspondingly.

2. In examination of the question of standards for the admissible deviation of frames and plating from true circular shape, let us dwell briefly on the method of measurement of the magnitude and forces of these deviations. Measurement of the initial bending of the plating in the span between frames can be readily made with the aid of a straight edge provided with a slide at the middle which directly determines any sag of the midpoint of the plating with respect to the frames. After a series of such measurements have been made at intervals of 0.1 to 0.2 of the distance between the frames, it will be possible with sufficient accuracy to get the shape and magnitude of the initial sag of the plating in its central cross-section.

The problem of finding the magnitude and shape of the initial deflection of the frames, and of its subsequent resolution into simplest components, is solved as follows:

Let $R_\alpha = f(\alpha)$ be a function representing the line where the frame is secured to the plating, expressed in polar coordinates, for which an arbitrary point O_1 is taken as a center coinciding insofar as possible closely with the true center of the circumference of the frame, (Figure 57). This function may be represented in the form of the following series:

$$R_\alpha = r_0 + a_1 \sin\alpha + b_1 \cos\alpha + a_2 \sin 2\alpha + b_2 \cos 2\alpha + a_3 \sin 3\alpha + b_3 \cos 3\alpha + \ldots \qquad [20]$$

For determination of the geometric meaning and the magnitude of the coefficients of this series, we multiply it in succession by $d\alpha$, $\sin\alpha\, d\alpha$, $\cos\alpha\, d\alpha$, $\sin 2\alpha\, d\alpha$, $\cos 2\alpha\, d\alpha$, etc., and integrate in the limits from 0 to 2π.

Multiplying by $d\alpha$ and integrating, we get:

$$r_0 = \frac{1}{2\pi}\int_0^{2\pi} R_\alpha \, d\alpha$$

Since the integral

$$\int_0^{2\pi} R_\alpha \, d\alpha$$

expresses the length of the circumference of the frame, the value r_0 therefore represents the radius of a circle (average radius of the frame) having a length equal to the length of this circumference.

Multiplying the series by $\sin\alpha d\alpha$ and by $\cos\alpha d\alpha$ and integrating, we get:

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} R_\alpha \sin\alpha \, d\alpha$$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} R_\alpha \cos\alpha \, d\alpha$$

The terms of the series $a_1\sin\alpha$ and $b_1\cos\alpha$ express translational displacements of the circle in mutually perpendicular directions of the magnitudes a_1 and b_1 respectively (Figure 58).

The displacements of the circle correspond to the shift of the initially arbitrarily chosen center point O_1 to the true center O of the frame circle with radius r_0.

Multiplying the series by $\sin 2\alpha d\alpha$ and $\cos 2\alpha d\alpha$ and integrating, we get:

$$a_2 = \frac{1}{\pi}\int_0^{2\pi} R_\alpha \sin 2\alpha \, d\alpha; \qquad b_2 = \frac{1}{\pi}\int_0^{2\pi} R_\alpha \cos 2\alpha \, d\alpha$$

The terms of the series $a_2 \sin 2\alpha$ and $b_2 \cos 2\alpha$ express the deflection of the frame (Figure 59, a and b) and the sum of these terms expresses the deflection (Figure 59 c) as determined by:

$$f = C_2 \sin(2\alpha + \epsilon_2) \qquad [21]$$

This deflection has two waves ($n = 2$); the frame is elliptical in shape, with the major axis, determined by the angle $\epsilon_2 = 1/2(\pi/2 - \text{arc tan } b_2/a_2)$, whereby, the largest value of the deflection of the frame is found to be

equal to:

$$C_2 = \sqrt{a^2 + b^2} \qquad [22]$$

Similarly, the coefficients of subsequent terms of the series, Equation [20] can be found, and the components of the deflection that they determine, using the following general expressions:

$$\left.\begin{aligned}
a_n &= \frac{1}{\pi}\int_0^{2\pi} R_\alpha \sin n\alpha d\alpha \\
b_n &= \frac{1}{\pi}\int_0^{2\pi} R_\alpha \cos n\alpha d\alpha \\
{}_n &= \frac{1}{n}\left(\frac{\pi}{2} - \text{arc tan}\,\frac{b_n}{a_n}\right) \\
C_n &= \sqrt{a_n^2 + b_n^2}
\end{aligned}\right\} \qquad [23]$$

where n is the number of waves of the considered component of the deflection shape of the frame.

The computation of the coefficients of the series, Equation [20], using the expressions obtained, must be made with the aid of Table 17.*

In Column I of Table 17 the numbers of subdivisions of the frame circle into equal parts are entered (Figure 57); Column II contains the distances from these points to the center of reference of the frame circle O_1. In the succeeding columns of Table 17 the values of the column headings are entered which occur in the expressions for the coefficients of the series, Equation [20].

Having designated the sums of the values of the columns by the symbol $\sum$ with a subscript identifying the column number, we get the following expressions for the computations of the coefficients of the series [20].

---

*The table was made for computation of the first five terms of the series [20]; for the calculation of subsequent terms of this series, the number of columns in the table must be increased correspondingly.

$$r_0 = \frac{1}{2\pi}\int_0^{2\pi} R_\alpha \, d\alpha = \frac{1}{2\pi}\frac{2\pi}{m}\sum_{II} = \frac{1}{m}\sum_{II}$$

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} R_\alpha \sin\alpha \, d\alpha = \frac{1}{\pi}\frac{2\pi}{m}\sum_{IV} = \frac{2}{m}\sum_{IV}$$

$$b_1 = \frac{2}{m}\sum_{VI}$$

$$a_2 = \frac{2}{m}\sum_{VIII}$$

$$b_2 = \frac{2}{m}\sum_{X}$$

3. If the found deviations from true circular shape of the frames and hull plating exceed the standards, specifications and production tolerances established for them, and if these deficiencies can not be eliminated by subsequent correction, all such zones of the hull must be reinforced in correspondence to the requirements of the formulas used for calculation and given in Section 23.

The design stress used in these formulas must be taken as equal to the stress at the design depth of submersion of the hull, while the limiting stress must be taken as equal to the yield point of the hull material.

When frame reinforcements are being made, what was said in Section 23, Paragraph 6, with respect to the unfavorable effect of the inwardly bent portion of the frame not only upon the strength of the frame itself, but also on the strength of the plating adjacent to it, must be taken into account. The necessity of reinforcement of the hull frames will be encountered in practice in exceptional cases only, principally in the region of incorrect concave bending. For reinforcement of the frame in this case, a belt or strap must be placed on the adjacent plating, to increase the moment of resistance of the section of the frame with respect to the compressed edge.

Mounting of a superimposed sheet on the concave side of incorrectly bent plating, and filling of the space between this sheet and the plating with water-proofing material (cement, red lead, etc.), must be considered as the best type of structural reinforcement of the plating in the

spans between frames.

In the case of a small initial incorrect bending of the plating, the reinforcement may be limited to the installation of longitudinal or transverse stiffeners between the frames.

25. FRAME REINFORCEMENT IN FILLETTED AREAS.

1. In the frames of the pressure hull, the necessity for fillets,* i.e. for local depressions and buckles of oval shape, occurs in the region where the propeller shafts pierce the pressure hull and in the region of the torpedo loading hatches.

Reinforcement of fillets in the region where the propeller shaft passes through the hull requires an adequate strengthening of the profile (section) of the frame, calculated for the action of a compressive force and of a bending moment in the frame section equal to

$$S = p_p r; \qquad M = S \times f; \qquad [24]$$

where p_p is the intensity of pressure on the frame at the design depth of hull submersion; r is the radius of the circumference of the frame; f is the greatest height of the fillet.

For fillets projecting beyond the outlines of the frame, the required local strengthening of the latter can be achieved easily by installation of gussets notched for the passage of the shaft. The inner edge of the gussets, suitably reinforced, will resist the thrust; this inner edge must be calculated for the action of a compressive force only equal to

$$P = p_p \times \overline{OO}_1 \qquad [25]$$

where $\overline{OO}_1$ is the distance between the center of the frame circumference and the center of the fillet circumference (Section 13).

In calculation of the strength of the reinforcement of a fillet with respect to the action of the forces determined by the above expressions, the maximum tensile stress must not exceed the yield point of the material and complete stability of all reinforcement connections must be guaranteed.

2. The height of the fillets in the pressure hull frames in the vicinity of the torpedo loading hatch changes along the length of the hatch, increasing in the direction of the entrance opening of the hatch. Reinforcement of low fillets can be made by local strengthening of the frame section

*Editor's Note: The Russian uses a term which may mean fillet, rounding off, recess, chamfer; these are obviously fillets in some cases and convex structures in others, faired as strength or space limitations require.

at its constant height, calculated for the application of an axial force and bending moment as determined by the expression in Equation [24]. For high fillets, reinforcement of this type is structurally difficult to realise; therefore, such fillets must be reinforced by removable rectilinear braces, mounted between the ends of the cut principal frame (Figure 60). In this case, the portion of the frame forming the fillet may have a reduced cross-section, corresponding to the radius of the circle of the fillet concerned. The stability of the brace must be calculated for its longitudinal compression force equal to (see Section 13)

$$P = q_p l \times \overline{00}_1 \qquad [26]$$

where q_p is the design pressure corresponding to the design depth of hull submersion; l is the distance between frames; $\overline{00}_1$ is the distance between the center of the frame circle and that of the fillet circumference.

The structural design of such a reinforcement of a fillet must satisfy the following requirements:

a) the axis of the strut must be rectilinear and practically coincide with the chord connecting the centers of gravity of the sections of the principal frame;

b) the compressive force must be transmitted into the strut not through the bolt connections but directly by the tightly fitted contact of the surfaces of the supporting structures;

c) the possibility of replacement of removable struts by corresponding expansion members designed for reinforcement of the fillets during testing of the hull in drydock by internal water pressure must be provided for.

26. HULL REINFORCEMENT AT CUT OPENINGS.

1. An examination of the conditions which lead to stress concentrations in areas where openings are cut in the cylindrical plating of the pressure hull of submarines leads to the conclusion that the magnitude and character of the distribution of such concentrations must remain the same as in the case of openings in a flat plate, subjected to compressive forces in mutually perpendicular directions. Having this in mind, it is possible for establishment of design and dimensions for the reinforcement of openings in the plating of submarine pressure hulls to use the corresponding data for reinforcement of openings in the plane connections of the hulls of surface vessels.*

*Shimansky, Yu. A.: "Designing of Discontinuous Hull Connections", State Publishing House for Shipbuilding, 1948.

However, there must be considered additionally the action of forces, normal to the cylindrical plating (water pressure), absent in the case of an opening in the flat connection of surface vessel hulls.

2. Stresses in the longitudinal and the transverse sections of the cylindrical plating of submarine hulls are determined by the following expressions:

$$\sigma_1 = -\frac{q_p r}{t}; \qquad \sigma_2 = -\frac{1}{2}\frac{q_p r}{t};$$

where r is the radius of the pressure hull; q_p is the external design pressure on the plating; t is the thickness of the plating.

When a circular opening is cut in the plating, stress concentrations occur in the region of this opening; these stresses reach their maximum on the edge in the longitudinal and transverse sections taken through the center of the opening (Figure 61).

The stress on the edge of the opening in the longitudinal section, passing through the center of the cutout, is

$$\sigma_{1_{max}} = 3\sigma_1 - \sigma_2 = -3\frac{q_p r}{t} + \frac{1}{2}\frac{q_p r}{t} = -2.5\frac{q_p r}{t};$$

similarly in the transverse section,

$$\sigma_{2_{max}} = 3\sigma_2 - \sigma_1 = -3 \times \frac{1}{2}\frac{q_p r}{t} + \frac{q_p r}{t} = -0.5\frac{q_p r}{t}$$

From a comparison of the above expressions for stresses, it is evident that a circular opening in the plating causes an increase of the stresses therein only in the longitudinal section, whereby the coefficient of stress concentration at the edge of the hole is found to be equal to 2.5. In accordance therewith, reinforcement of such an opening must consist of the installation along its edges of transverse strap plates only, (Figure 62). The usual practice of installing a reinforcing plate embracing or surrounding the entire opening is not only unnecessary but favors an undesired attraction, to the region about the opening, of stresses acting in longitudinal sections of the plating.

3. In the determination of the thickness of reinforcing strap plates (Figure 62), there must be taken into account not only the magnitude of the coefficient of stress concentration found above (i.e. 2.5), but also the extent of the region of stress concentration, proportional to the diameter of the opening. The extent of the region of stress concentration or, consequently the length of the diameter of the opening, must be evaluated in relation to the thickness of the plating. It may be considered that if the ratio of the diameter of the opening to the thickness of the plating is of

an order of 10 - 15 or less, the excessive stresses in the region of the opening have such a localized extent that they cannot have any notable unfavorable effect on the strength of the plating. With the increase of the ratio of the diameter of the opening to the thickness of the plating, these excessive stresses begin to become more general in extent, at which the strength of the plating will prove to be correspondingly decreased. On the basis of what has been said above, the following expression for determining the thickness of reinforcing strap plates may be recommended (Figure 62):

$$t_1 = 2t\left[1 - 15\frac{t}{d}\right] = kt, \qquad [27]$$

where t_1 is the thickness of the strap plates; t is the thickness of the hull plating; d is the diameter of the opening; k is a coefficient.

The curve given in Figure 63 shows the change of the coefficient k in Equation [27] in function of the ratio of the diameter of the opening d to the thickness of the plating t. The limiting value of this coefficient, with $d/t = \infty$, equal to $k = 2$, corresponds to a coefficient of stress concentration equal to 3, i.e. to a value exceeding the largest obtained above in the longitudinal section of the opening which was equal to 2.5. Such an excess is justified by the fact that the presence of the reinforcing strap plates must change the distribution of stresses somewhat unfavorably in the region of the opening, in comparison with what it would be without strap plates, at which the coefficient of stress concentration was 2.5.

4. The water pressure upon the plating is equilibrated on the plating by the stress components acting in the longitudinal sections of the plating (Figure 64 a). For the equilibration of the hydraulic pressure absorbed by the cover of the cutout and by a certain region of the plating itself adjacent to the opening, it is necessary to have a supplementary reinforcement consisting of longitudinal beams mounted between the nearest frames (Figure 64 b). The design loading, acting on each of the beams, can be taken as a load distributed as a triangle on a segment of the beam of length d and equal to

$$Q = \frac{1}{2} q_p d^2$$

where q_p is the design water pressure and d is the diameter of the opening.

If the diameter of the opening equals the distance between the frames, the maximum bending moment in the sections of the beam will equal

$$m_{max} = \frac{1}{6} Ql = \frac{1}{12} q_p l^3, \qquad [28]$$

wherein l is the distance between the frames.

When reinforcing an opening for direct action of hydraulic pressure, a coaming about the opening may be used as one of the members for reinforcing the opening. At a loading q_p, corresponding to the design depth of submersion of the submarine, the maximum stresses in the reinforcing members should not exceed the yield point of the material.

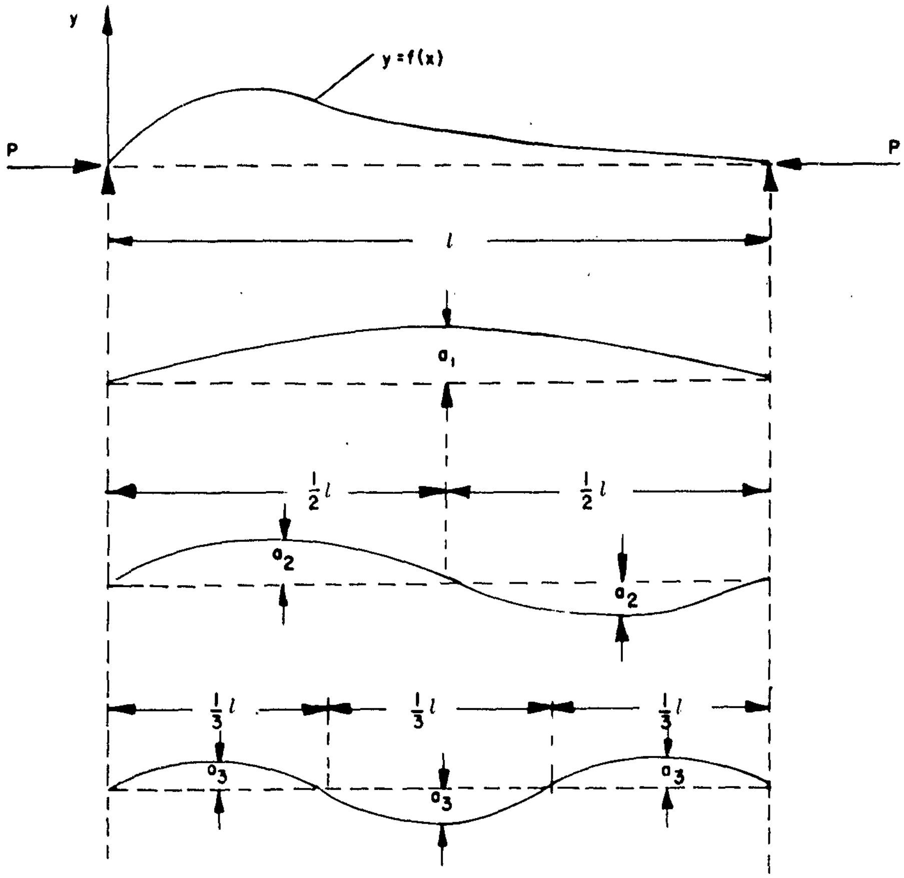


Figure 55

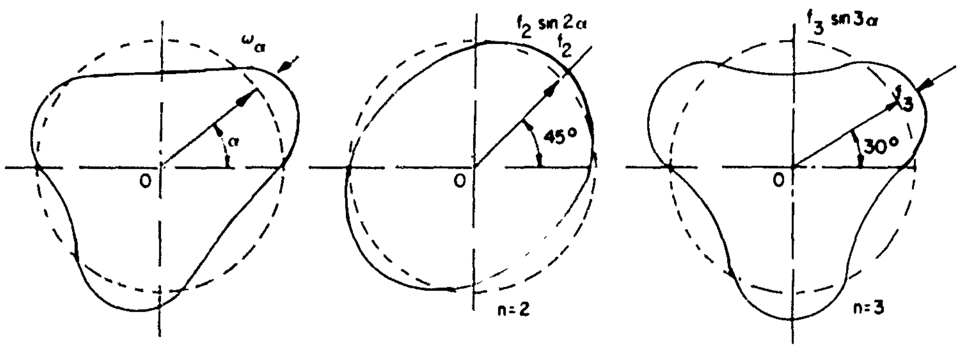


Figure 56

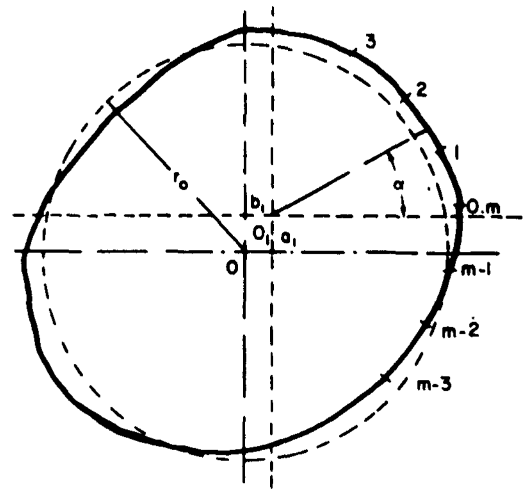


Figure 57

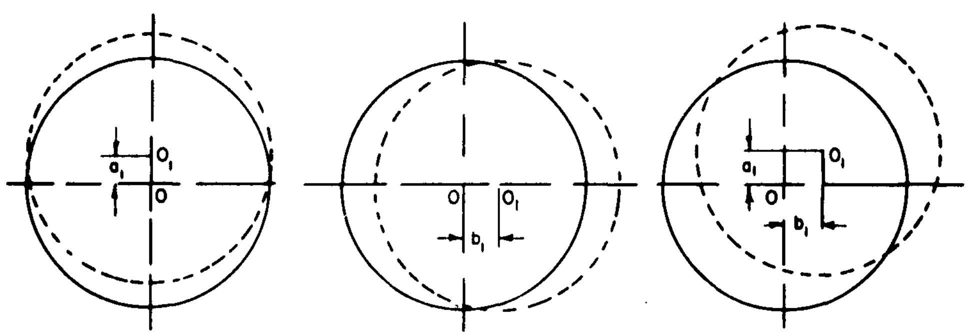
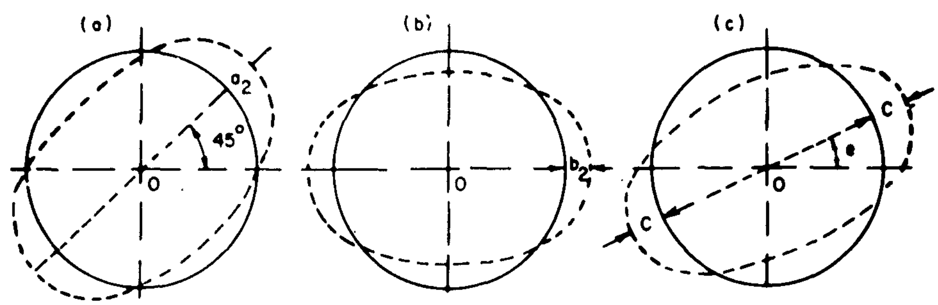


Figure 58



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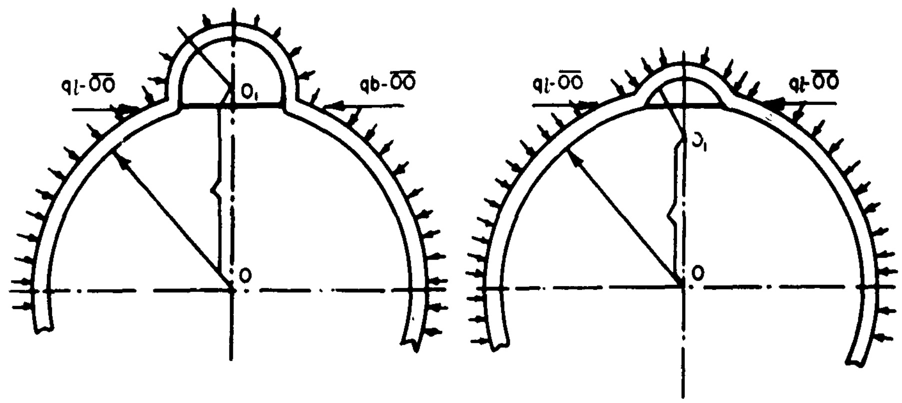

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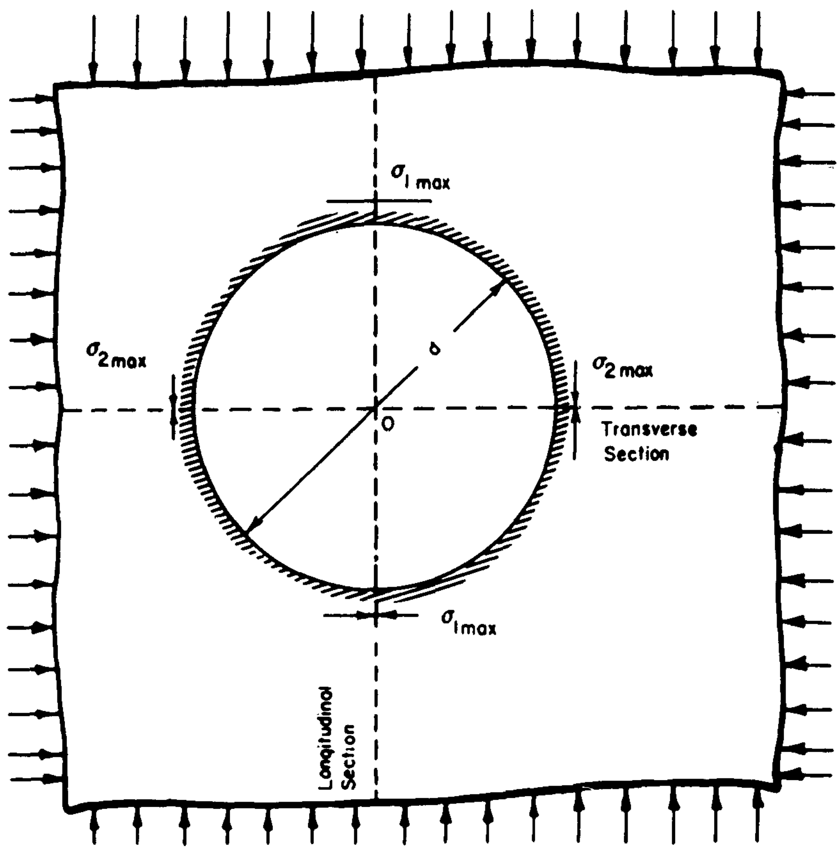

Figure 61

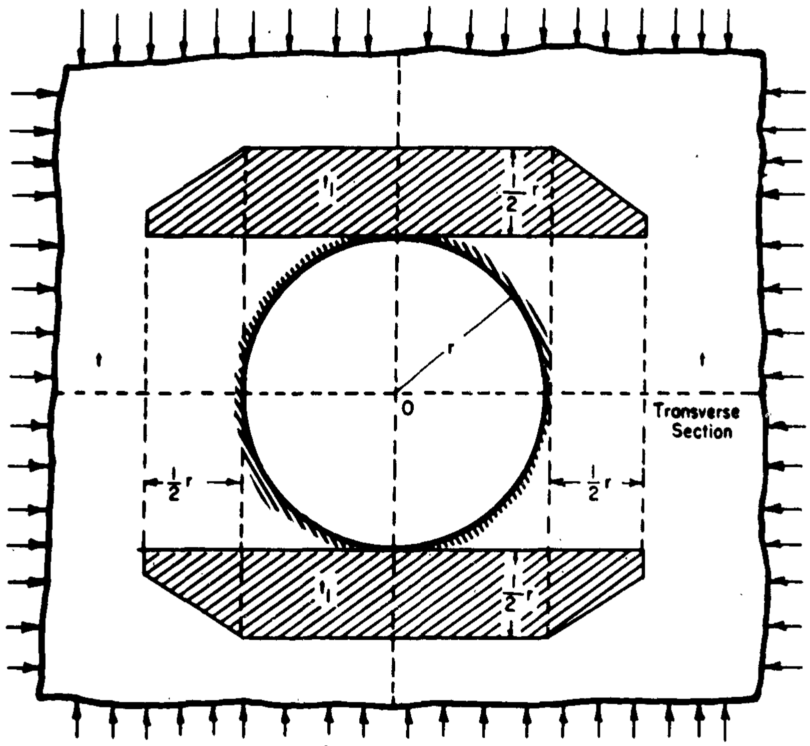

Figure 62

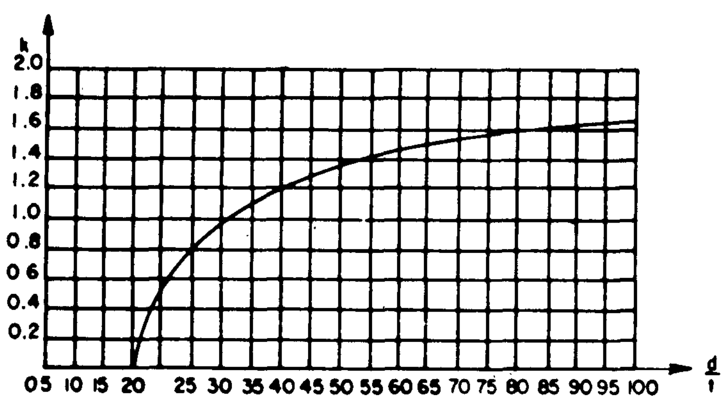

Figure 63

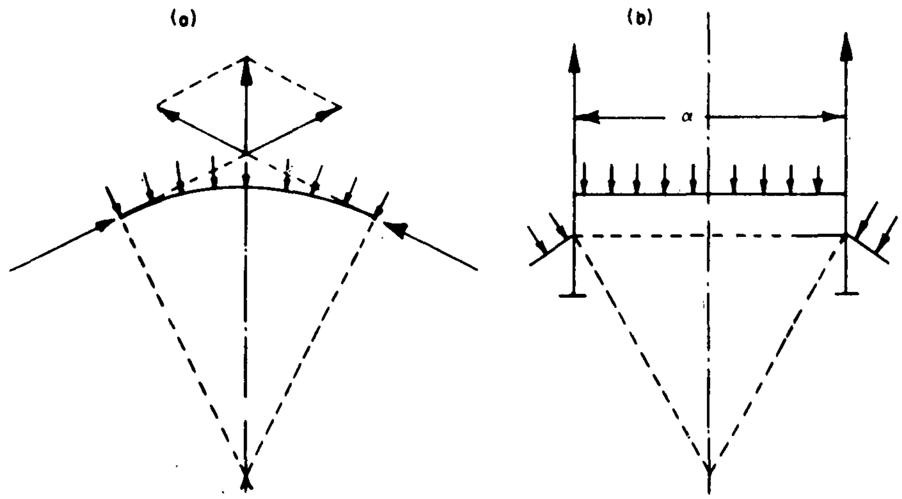

Figure 64

Table 17

I	II	III	IV	V	VI	VII	VIII	IX	X
Number of Section	R_α	$\sin\alpha$ $\sin\frac{2\pi}{m}\cdot I$	$R_\alpha \sin\alpha$ II·III	$\cos\alpha$ $\cos\frac{2\pi}{m}\cdot I$	$R_\alpha \cos\alpha$ II·V	$\sin 2\alpha$ $\sin\frac{4\pi}{m}\cdot I$	$R_\alpha \sin 2\alpha$ II·VII	$\cos 2\alpha$ $\cos\frac{4\pi}{m}\cdot I$	$R_\alpha \cos 2\alpha$ II·IX
0 1 2 . . . m−2 m−1	R_0 R_1 R_2 . . . R_{m-2} R_{m-2}								
	ΣI	—	ΣIV	—	ΣVI	—	ΣVIII	—	ΣX

CHAPTER VII

STRENGTH ANALYSIS OF DECK STRUCTURES

27. GENERAL FOUNDATION OF STRENGTH ANALYSIS OF DECK STRUCTURES AND THEIR VARIOUS TYPES.

1. Rigid or stable deck structures of submarines are the superstructures mounted on the pressure hull, possessing the same resistance to external water pressure as does the entire pressure hull. Therefore, the general foundations for calculation of strength of deck structures are the same as those stipulated for the pressure hull. Thus, such stable deck structures must be analyzed with respect to both strength and stability, by taking the magnitude of the <u>design</u> load as given by Equation [10] of Section 3, which defines the design loading for the pressure hull. Determination of the stresses in the sections of a deck structure, the determination of its critical loading and the establishment of standards or norms with respect to dangerous stresses must be carried out taking into account the construction and shape of the deck structure involved, i.e. with respect to the particular type involved.

According to the outline, stable deck structures are subdivided into two basic types: (a) <u>circular</u> or round deck structures, (b) <u>oval</u> or elliptical deck structures.

2. Calculation of the stability of a <u>circular</u> deck structure (either perpendicular or horizontal) differs in no way from that used for calculation of circular submarine pressure hulls. A certain difference consists only in the thickness of the plating, usually determined by the requirement of guaranteeing a combat strength* sufficiently high for deck structures. In virtue of this requirement, the thickness of the plating of deck structures may prove to exceed notably the calculated thickness for strength and stability of the plating. Taking this into account, the distance between the transverse ribs, stiffeners of the deck structure, determined according to conditions of its stability, may be increased correspondingly. In a number of cases, this circumstance may lead to a design of deck structures without stiffeners; these structures possess a certain advantage insofar as the internal disposition is concerned.

3. <u>Oval</u> or elliptical shaping of deck structures is more favorable from the point of view of dimensions as compared to circular structures. However, such oval designs are less advantageous from the point of view of

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*Editor's Note: The Russian uses here the term which, literally translated, signifies "tenacity of life"; this the translator suggests may be "combat survival". In either case, it apparently must be a high safety factor.

resistance to water pressure. Yet this disadvantage of oval design of deck structures, compared to circular types, decreases greatly as the ratio of the height of the deck structure to its transverse dimensions diminishes, due to the participation, in the resistance of the deck structure, of the transverse compression of its rigid end structures (cover and the portion of the pressure hull, adjacent to the base of the deck structure). This circumstance, as well as the increased thickness of the plating of deck structures, justify the use in practice of oval deck structures with a vertical axis.

28. VARIOUS DESIGNS OF OVAL DECK STRUCTURES.

1. In practice, the following two varieties of shapes of oval deck structures are used:

(a) in one case, the curve of the transverse sections of the deck structure is made up of arcs of different circles, having common tangents at the points of their juncture (Figure 65);

(b) in the other case, the curve of the transverse sections of the deck structure is designed according to a true ellipse.

With the usual relations between the length and breadth of an oval deck structure, both these shapes insofar as dimensions are concerned, seem almost equivalent; however, a certain advantage prevails in favor of deck structures made up of circular arcs. From the production point of view, arcs of true circles appear the more favorable shape for designing an oval deck structure. However, the technological advantages of such a shape become of small significance in mass production of submarines.

From the point of view of the resistance of oval and elliptical deck structures with respect to water pressure, a true elliptical shape appears more advantageous than an oval design consisting of circular arcs.

Such a conclusion may be drawn from the consideration that at the points of juncture of the arcs of the various circles there results an interruption of the continuity in the curvature constituted by these arcs in the section of an oval deck structure.

2. In Figure 65 the shape of an oval deck structure is shown, constituted by arcs of true circles of two different radii. The shape and dimensions of such a deck structure are defined by the following relationships existing between the five quantities:

$$R - b = (R - r)\cos\alpha$$

$$a - r = (R - r)\sin\alpha$$

where R is the radius of the circles forming the side walls of the deck structure; r is the radius of the circles forming the fore and aft walls of the deck structure ($r < R$); a, b are the half-length and half-width of the

deck structure; α is the angle determining the point of junction of circles of radii r and R.

Using these functions, we can get the following expressions for the radii r and R;

$$R = \frac{a \cos \alpha - b(1 - \sin \alpha)}{\sin \alpha + \cos \alpha - 1} \qquad [1]$$

$$r = \frac{b \sin \alpha - a(1 - \cos \alpha)}{\sin \alpha + \cos \alpha - 1} \qquad [2]$$

The quantities a and b contained in Equations [1] and [2] determine the over-all dimensions of the deck structure, i.e. its length and breadth. The angle α determines the shape of the deck structure within the limits of the specified over-all (or clearance) dimensions. In Figure 66 two extreme forms of deck structures are shown which correspond to the principles of their design here under investigation.

Curve I shows the fullest shape obtained with $\alpha = 0$ and correspondingly $R = \infty$ and $r = b$. Curve II shows the sharpest or most pointed shape, obtained with $\alpha = \arcsin [2ab/(a^2 + b^2)]$ and correspondingly $R = 1/2b[(a^2 + b^2)]$ and $r = 0$. This shape is made up of two circles of radius R. The broken line shows an elliptical deck structure.

For such an elliptical deck structure design, the largest value (R) and the smallest value (r) of radii of curvature of its surface are determined by the following well-known formula:

$$R = \frac{a^2}{b}; \qquad r = \frac{b^2}{a} \qquad [3]$$

29. STRENGTH ANALYSIS OF OVAL DECK STRUCTURES.

1. If the walls of an oval deck structure were not supported by rigid end structures (the roof or cover of the deck structure and the portions of the pressure hull adjacent to it), they would necessarily undergo bending deformation under the action of the external water pressure; this would increase the length of the deck structure while decreasing its breadth.

The rigid end structures of a deck structure impede deformation of the walls, at the expense of shearing forces, produced along the line of juncture of the wall to the rigid structures. Shearing forces, which hinder a large bending of the wall, will increase the compressive forces acting in its sections, i.e. they will promote the increase of rigidity and resistance of the wall with respect to the compressive loading acting on it. In this case, the resistance of the wall of the deck structure to an external compressive load will be determined not by the magnitude of the

bending stresses occurring in its sections, but by the magnitude of the direct compressive stresses occurring in these sections which may disturb the general or local stability of the wall. Therefore, the stability of an oval deck structure, as well as that of a round or circular deck structure, should be analyzed for stability of its walls, taking into account the effect on their critical loading of the magnitude of the largest compressive stresses acting in the sections of the wall at this loading.

2. With a correct elliptical shape of the transverse sections of an oval deck structure, the stresses generated in its walls by bending will be small in comparison to the membrane stresses and therefore, their effect on the stability of the walls may be disregarded. In this case, making the error on the safe side, it is possible to determine the critical loading on the walls of a deck structure as is done for a circular cylindrical shell, taking the radius of this shell as equal to the largest radius of curvature (R) of the elliptical section of the deck structure as determined by Equation [3].

The largest compressive stress in the longitudinal section of the wall of the deck structure corresponding to the radius R, which ought not to exceed 80% of the yield point of the material, must be taken as equal to:

$$\sigma = 1.1\, q_{\text{в}} \frac{a^2}{bt} \qquad [4]$$

where $q_{\text{в}}$ is the critical loading, determined by the method set forth in Section 6; t is the wall thickness of the deck structure.

3. The order of strength analysis of an elliptical oval deck structure can be used also for the calculation of strength of an oval deck structure, having the form of conjugate (connected) arcs of true circles. However, with such a deck structure design, the wall stresses generated by bending will significantly exceed the stresses produced by bending in the walls of an oval deck structure of correct elliptical shape, and consequently, the effect of these stresses on the stability of the walls of the deck structure must be more substantial. In addition to the foregoing, in this case the region of the wall having the largest radius of curvature R and subject to the action of the greatest compressive membrane stresses will be considerably larger than is true for the elliptical shape of oval deck structures.

Both these unfavorable circumstances can be taken into account by a certain reduction of the standard, established above for the magnitude of the compressive stresses as determined by Equation [4]. With the usual over-all dimensions of oval deck structures 2 m high, this standard must be reduced as much as 50% of the yield point of the material.

4. The shearing forces generated along the connecting lines of the deck structure walls to its end structures play a decisive part in the performance and resistance of oval deck structures. The magnitude of the shearing force τ, whose general character of variation is shown in Figure 67,

must be directly proportional to the height of the deck structure. As the deck structure height is increased, the shearing forces and the shear stresses corresponding to them in the transverse sections may limit the resistance of such an oval deck structure. However, with the usual over-all dimensions of oval deck structures, ranging in height to 2 m, the shearing forces and the corresponding shear stresses must not be a cause for apprehension, provided that the limitations on the magnitude of the largest membrane stresses in the longitudinal sections of the deck structure, indicated above, be satisfied.

5. A complete analytical investigation of the deformations of an oval deck structure consisting of true circular arcs was made by V. V. Novozhilov.* Such an investigation is characterized by very cumbersome methods of calculation whose practical value is decreased further by their lack of a check on the stability of such deck structures. For the case of an oval deck structure of true elliptical shape, V. V. Novozhilov developed the simple approximation formulas cited below, which determine the membrane stresses in the sections of the deck structure.

The greatest normal stress in the longitudinal section of the deck structure:

$$\sigma_1 = q\frac{a^2}{bt} \qquad [5]$$

The greatest normal stress in the transverse sections of the deck structure:

$$\sigma_2 = \frac{qb}{t}\left[1 + \frac{3}{8}\frac{h^2}{b^2}e^2\right] \qquad [6]$$

The greatest tangential stress in the walls of the deck structure:

$$\tau = \frac{3}{4}\frac{qh}{t}\left[\frac{a}{b} - \frac{b}{a}\right] \qquad [7]$$

where a, b are the half-length and half-width of the deck structure; h is the height of the deck structure; t is the wall thickness; $e^2 = 1 - b^2/a^2$.

Example:

To determine the limit strength (resistance) of an oval deck structure of elliptical section, having the following dimensions:

*Novozhilov, V. V. and Slepov, B. I.: "Calculation of the Stresses in the Structures of Submarine Hulls with Consideration for the Effect of Transverse Bulkheads", Oborongiz, 1945.

length:	2a = 350 [cm]
breadth:	2b = 200 [cm]
height:	h = 250 [cm]
thickness of walls:	t = 4 [cm]
yield point of deck structure material:	σ_T = 3000 [atm]

The largest radius of curvature (at the extremities of the minor axis) by Equation [3] is:

$$R = \frac{a^2}{b} = \frac{175^2}{100} = 306 \ [\text{cm}]$$

The structural stability of the deck structure must be determined by the expressions in Section 6, taking the following values in them:

$$r = R = 306 \ [\text{m}]$$

$$l = h = 250 \ [\text{cm}]$$

$$t = 4 \ [\text{cm}]$$

$$\gamma = r/l = 306/250 = 1.22$$

$$\delta = t/r = 4/360 = 13.1 \times 10^{-3}$$

Since the values obtained for γ and δ are found to exceed the limits given in Table 8 and curves of Figure 9, the theoretical critical pressure $q_э$, must be computed directly by Equation [17] of Section 6, assuming an integer n = 9 from Table 7 with $\gamma/\delta = 1.22/13.1 \times 10^{-3} = 93$.

If we substitute into Equation [17] n = 9, $\delta = 13.1 \times 10^{-3}$, $\alpha = \pi\gamma = 3.83$ and take $X = 0$, it is found that $q_э = 60$ [atm].

The correction factor η_1 according to Equation [24] of Section 7 is:

$$\eta_1 = \left(1 - \frac{0{,}3}{t}\right) = \left(1 - \frac{0{,}3}{4}\right) = 1$$

The theoretical critical stress by Equation [18] of Section 6 is:

$$\sigma_э = 1.1\,\eta_1 \frac{q_э r}{t} = 1.1 \times 1 \times \frac{60 \times 306}{4} = 5050 \ [\text{atm}]$$

The correction factor η_2 according to Table 10 for steel having a yield point of 3000 atm is:

$$\eta_2 = 0.61$$

The actual critical pressure according to Equation [19] of Section 6 is:

$$q_1 = \eta_1 \eta_2 q = 0.61 \times 1 \times 60 = 36.6 \text{ [atm]}$$

The stress corresponding to this pressure is:

$$\sigma_{1э} = \eta_2 \sigma_э = 0.61 \times 5050 = 3080 \text{ [atm]}$$

Since the stress corresponding to the critical pressure exceeds the standards specified for admissible stress ($0.8\,\sigma_T = 0.8 \times 3000 = 2400$ atm), the limit strength of the deck structure being investigated must be taken as equal, not to 36.6 [atm], but as

$$36.6 \times \frac{2400}{3080} = 28.5 \text{ [atm]}$$

The maximum shear stress in the sections of the deck structure according to Equation [7] of this paragraph (Section 29, Paragraph 5) is:

$$\tau = \frac{3}{4}\frac{qh}{l}\left(\frac{a}{b} - \frac{b}{a}\right) = \frac{3}{4} \times \frac{28.5 \times 250}{4}\left(\frac{175}{100} - \frac{100}{175}\right) = 1580 \text{ [atm]}$$

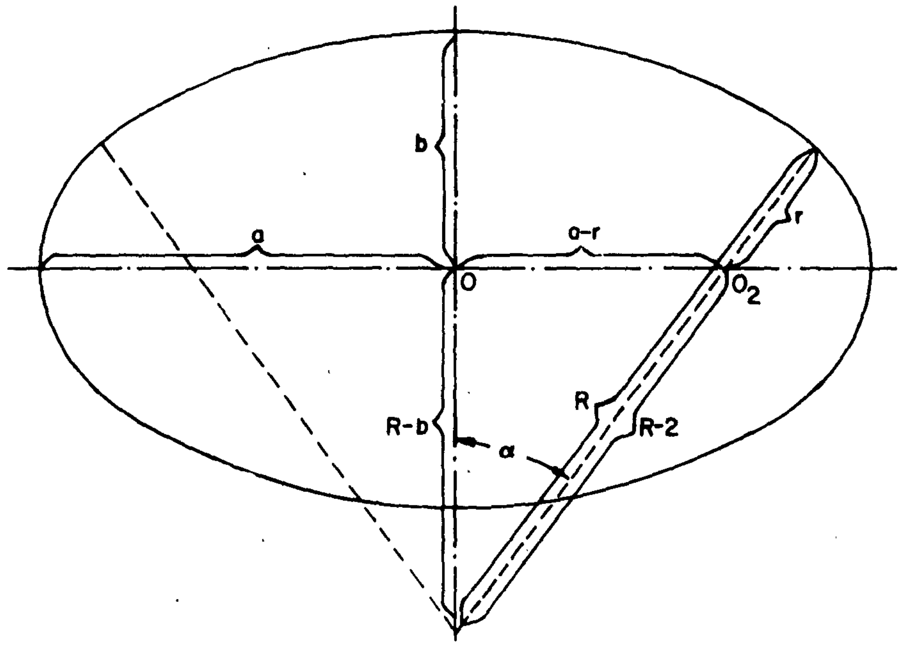

Figure 65

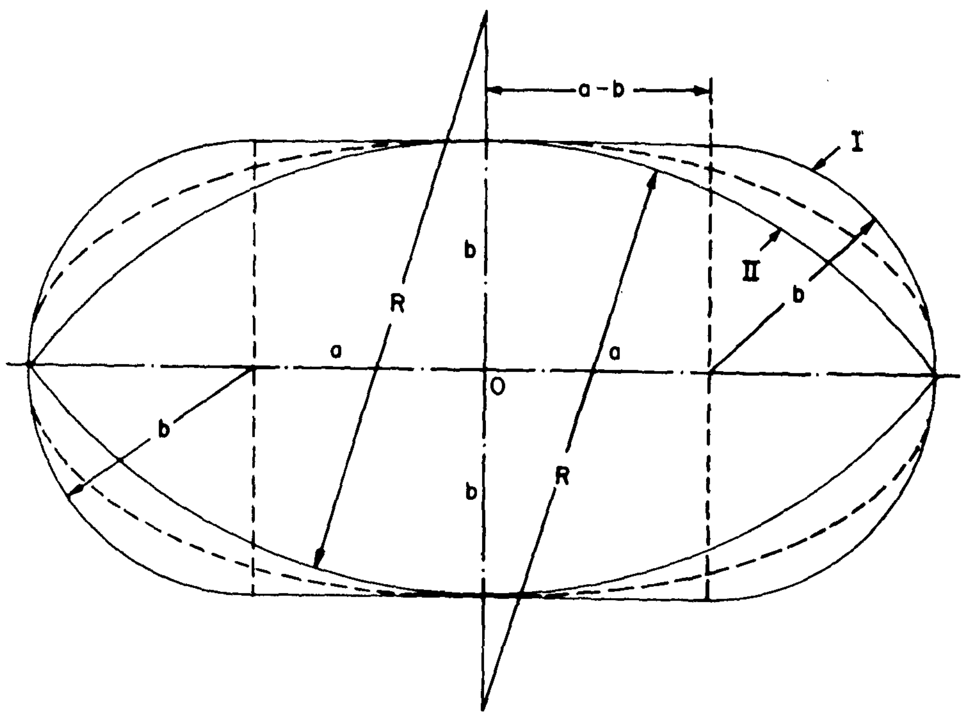

Figure 66

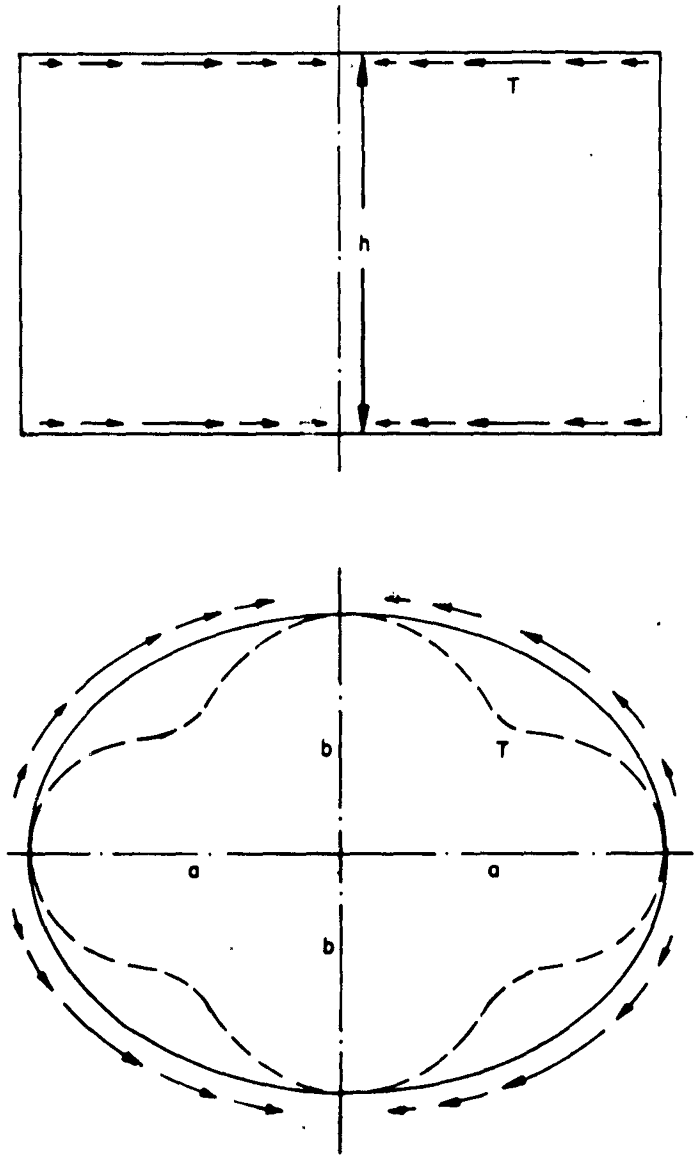

Figure 67

CHAPTER VIII

ANALYSIS OF THE DRYDOCKING OF SUBMARINES

Owing to the comparatively great general and local strength of submarine hulls, no difficulties are encountered in drydocking them such as are encountered in the case of surface vessels where special investigations and special reinforcements are often necessary. However, this does not eliminate the necessity for carrying out calculations for drydocking of all new types of submarines, with the objective of establishing thus the simplest and most expedient system of drydocking them, and likewise for emergency drydocking where an unusual distribution of keel blocks and crib-work* beneath submarine hulls may prove necessary.

Below is set forth the general order for carrying out the analysis of the general and local strength of a submarine hull when drydocking it.

30. CONSTRICTION OF THE DIAGRAM OF BENDING MOMENTS.

The hull of a submarine, lodged in the drydock, represents a beam of variable section, equilibrating upon itself the distributed forces of the weight and the concentrated reactions of the keel blocks and crib-work.

For determination of the bending moment in any section of the hull, i.e. of the moment of all the forces on one side of the section under investigation, it is required to know the magnitude and distribution of the above indicated forces of the weight and of the reactions of crib-work and keel blocks. The magnitudes and distribution of forces of weight of the submarine will be considered as known and as given in the form of a weight curve, whose ordinates at a certain scale, represent the weight of the submarine, acting per unit length [t/m]. With the weight curve available, it is easy to calculate and plot the curve of the moments of the weight forces along one side of the section (Figure 68); the computations associated with this operation are tabulated in the usual manner, as shown in Table 18.

Using the data in the last line of Table 18, the curve for the moments of weight (I) must be constructed, whose ordinates, measured according to a certain scale, will give the value of the moment of the weight forces of that portion of the submarine located toward the left (i.e. toward the stern) of the section being examined (Figure 68).

For plotting the moment curve of the forces of reaction of the drydock, it is required first of all to find the magnitude of these reactions at a given distribution of keel block and crib-work along the length of the submarine and for a given design of these objects. The moment curve

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*Editor's Note: The Russian here uses the word "cage," but must mean supporting crib-work designed to shore up the hull in drydock.

of the drydock reactions will be given as a broken line whose individual linear parts are segments of straight lines representing the value of the moment for each keel block, group of keel blocks or crib-work, separately. For the construction of this broken line, it is necessary to lay out consecutively, on a perpendicular passing through the tip of the bow of the submarine, the values for the moments of the reaction forces (m_i) relative to the tip of the bow; through the points obtained, straight lines are then drawn as shown in Figure 68. The values of the indicated moments m_l,* necessary for construction of the moment curve of the drydock reaction forces, are found in the last column of Table 19 and serve for computation for the reaction of keel block and crib-work.

Having the curve for the weight moments for the submarine, I, and the curve of the drydock reaction moments, II, we can then construct the curve for the bending moments with respect to ordinates equal to the differences of the ordinates of curves I and II.

31. DETERMINATION OF DRYDOCK REACTIONS.

In view of the comparatively great rigidity of submarine hulls, it is possible in the determination of keel block and crib-work reactions to consider, with sufficient practical accuracy, the hull as absolutely rigid. Using such an assumption, determination of the reactions of the drydock is greatly simplified and may be carried out according to the following computational scheme.

1. Notation (Figure 68):

L = the length of the submarine;

x_i = distance of crib-work, keel block or group of keel blocks from the bow perpendicular;

$l_1\ l_2$ = vertical deflection of bow and stern as a result of compression of crib-work and keel blocks;

$\alpha = \dfrac{l_2 - l_1}{L}$ = angle of inclination of the submarine resulting from compression of crib-work and keel blocks;

$y_i = l_2 - \ x_i$ = vertical deflection of a section at a distance x_i from the bow, resulting from compression of crib-work and keel blocks.

S_i = area of a horizontal section of a keel block, group of keel blocks or of crib-work;

k_i = coefficient of rigidity of a keel block or of crib-work;

*Editor's Note: Possibly in error; m_l not shown in Figure 68, but m_1, m_2 and m_3 are.

$p_i = k_i y_i$ = specific pressure applying on a keel block or on crib-work;

$P_i = S_i p_i$ = reaction of a keel block, group of keel blocks or of crib-work;

D = weight of the submarine;

a = distance of the center of gravity from the bow perpendicular.

2. Determination of the coefficient of rigidity k_1. The coefficient of rigidity k_1 of a keel block or cage is measured in units of the specific pressure and represents that value of the specific pressure at which the setting (compression) of a keel block equals the assumed unit of length (for example, the specific pressure is measured in kg/cm^2 whereas the compression is measured in cm).

The coefficient of rigidity is determined in dependence of the design and construction of the keel block as follows.

Let the keel block be constructed of pine beams of average height h_c and of oak beams of average height $h_ð$* (Figure 69); let the compression for pine and oak be represented by curves, where, along the abscissa are marked off specific pressures p, and along the ordinate the corresponding relative compressions i.

Let us designate by E_c and $E_ð$ the moduli of deformation for pine and oak, representing the ratio of the specific pressure to the corresponding relative compression ($E = p/i$). The values of the modulus of deformation for timber depend, as is evident from the slope of the curve shown in Figure 69, not only on the type of wood, but also on the value of the specific pressure. Therefore, in the determination of the coefficient of rigidity of keel blocks, we must establish the magnitude of the specific pressure for which it is to be found.**

Either by use of reference data or by use of experimental curves for wood compression*** are found the values for the moduli of deformation E_c and $E_ð$ which make it possible to determine the general compression of the

---

*Editor's Note The Russian word for "oak" starts with the character ð when written in script letters.

**If, as a result of the calculation, it is found that the specific pressure obtained differs very appreciably from that assumed, it is necessary then to repeat the calculation, assuming the value of the specific pressure found in the first calculation.

***In the absence of experimental data, we must take E_c = 500 to 700 [atm] and $E_ð$ = 5000 to 7000 [atm], depending on the degree of saturation with water (water soaking) of the keel blocks (higher values for dry wood).

keel block at a specific pressure p by the expression:

$$y = h_e i_e + h_\partial i_\partial = h_e \frac{p}{E_e} + h_\partial \frac{p}{E_\partial} = p \frac{E_\partial h_e + E_e h_\partial}{E_e E_\partial}$$

Consequently, the coefficient of rigidity will be:

$$k = \frac{E_e E_\partial}{E_\partial h_e + E_e h_\partial} \quad [1]$$

Using Equation [1] and knowing the compression curves for oak and pine, it is possible, for a given keel block design, to plot the curve for the change of its coefficient of rigidity as a function of the value of the specific pressure.

3. Computation of the drydock reactions. The reaction of a keel block (or of crib-work) is determined according to the following obvious expression:

$$P_i = S_i p_i = S_i k_i y_i = S_i k_i (l_2 - \alpha x_i),$$

where S_i = the effective area of a section of the keel block; p_i = specific pressure for the compression of the keel block; k_i = coefficient of rigidity of the keel block at the specific pressure p_i; $y_i = (l_2 - \alpha x_i)$ = compression of the keel block, where x_i is its distance from the bow perpendicular.

The quantities l_2 and α which determine the general settlement of the hull, can be found from the two equations given below, expressing the conditions of equilibrium between the forces of the weight of the submarine and the reactions of the drydock, to wit:

$$\Sigma P_i = D$$

$$\Sigma P_i x_i = D \times a$$

Substituting in these equations the expression for P_i and solving them with respect to l_2 and α we shall get:

$$l_2 = \frac{C - Ba}{AC - B^2} D \quad [2]$$

$$\alpha = \frac{B - Aa}{AC - B^2} D \quad [3]$$

where

$$A = \Sigma S_i k_i; \quad B = \Sigma S_i k_i x_i; \quad C = \Sigma S_i k_i x_i^2$$

Computation of the reaction of the drydock according to the expressions cited is conveniently tabulated; see Table 19.

In column I are entered the numbers of the crib-work, keel blocks or groups of keel blocks on which the submarine is shored up in drydock.

In column II are entered the distances of the crib-work and keel blocks from the bow perpendicular x_i [m].

In column III are entered the portions of the areas of the sections of crib-work and keel blocks subjected to pressure S_i [m^2].

In column IV are entered the coefficients of rigidity of crib-work and keel blocks, determined by Equation [1]; k_i [kg/cm^3].*

In column V are entered the products of columns III and IV (III·IV), i.e. the products of S_i and k_i.

To get the result in [t/cm], these products are multiplied by 10 (the load on a keel block in [t] corresponds to the compression of the keel block on 1 cm).

In column VI are entered the products of columns II and V, i.e. the products $10S_ik_ix_i$.

In column VII are entered the products of columns II and VI, i.e. the products $10S_ik_ix_i^2$.

Adding the values in columns V, VI and VII we get the quantities A, B and C in Equations [2] and [3] which serve for computation of the values of l_2 and α. Having determined by these formulas l_2 and α, we fill in the following columns of the table, serving for determination of the reactions of the drydock.**

In column VIII are entered the products of α times the values of column II (α.II), getting the result in cm.

In column IX are entered the differences of the quantity l_2 and the values of column VIII (l_2 - VIII).

The values in column IX give the compression of the keel blocks y_i in cm; the last figure of this column gives the quantity l_2.

In column X are entered the products of the values in columns IV and IX (IV.IX), i.e. the values of the specific pressure on the keel blocks, $p_i = k_iy_i$ [kg/cm^2].

In column XI are entered the products of column III and X (III.X), i.e. the values of the reactions of the keel block $P_i = S_ip_i$ [t].

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*The coefficients of rigidity k_i must be determined for the first approximation by proceeding from the value of the specific pressure, obtained with the assumption of a uniform distribution of pressure along the entire area of keel blocks and crib-work, i.e. setting it equal to $p = D/10\sum S_i$ [kg/cm^2], where D is the weight of the submarine in tons and $\sum S_i$ is the sum of the numbers appearing in column III of the table.

**The quantity l_2 will be obtained in cm. The quantity α will be found at a value 100 times greater than its true value; by a further multiplication of α by x (in meters), we get the result in cm (column VIII).

In column XII are entered the products of values in columns II and XI (II.XI), i.e. the values of the moments of reaction of the keel blocks with respect to the bow perpendicular $M_1 = P_1x_1$ [t-m].

32. CHECKING THE GENERAL LONGITUDINAL STRENGTH OF THE HULL.

For checking the general longitudinal strength of a submarine, the maximum bending stresses of the hull must be determined and compared with the value of the admissible stresses. In determination of the bending stresses, in the portion of the length, limited by the rigid hull, in computing the moment of resistance of a section, only longitudinal connections of the pressure* hull need be considered, neglecting the contribution of the longitudinal connections of the outer hull. In this case, the moment of resistance of a section of the hull can be computed according to the following simplified formula:**

$$W = \pi r^2 t$$

where r = the radius of the circular hull in the section under investigation; t = thickness of hull plating in this section.

The greatest stresses in the hull plating will be:

$$\sigma_{max} = \pm \frac{M_{max}}{W} = \pm \frac{M_{max}}{\pi r^2 t} \qquad [4]$$

where M_{max} is the bending moment whose value is determined from the bending moment diagram.

The admissible stress must be taken in function of the value of the yield point of the plate material of the hull and also in function of the critical stress for stability of the plating. For this purpose, in establishment of the necessary reserve strength, we must consider the special conditions relative to drydocking submarines. In general, i.e. under normal drydocking conditions, the admissible stress may be taken as equal to 60% of the yield point of the plate material and as high as 60% of the critical stress with respect to the stability of the plating; the smaller value of stresses thus obtained should be taken.

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*Editor's Note: The Russians have here again changed their nomenclature. What was formerly termed in this work the "stable" or "solid" hull appears now to be termed the "rigid" hull; all apparently refer to the pressure hull.

**The moment of inertia of a section of a cylindrical shell with radius r and thickness t is:

$$I = \pi r^3 t$$

The critical stress with respect to stability of the hull plating can be found by the following approximation formula:

$$\sigma_{Kp} = \frac{Et}{r} \times \frac{1}{\sqrt{3(1-\mu^2)}} \approx 1.3 \times 10^6 \frac{t}{r} \text{ [atm]} \qquad [5]$$

33. CHECKING LOCAL HULL STRENGTH.

From the point of view of local hull strength, crib-work and keel blocks must be placed chiefly beneath the transverse bulkheads of the hull whose rigidity, relative to the forces of reaction of the drydock, appears completely ensured and therefore does not require any verification. In general, however, the local strength of the hull must be checked for (1) local bending of the hull plating between frames subjected to the application of a distributed load produced by reactions of the drydock and (2) bending of the frames, subjected to the action of concentrated forces, applied at their lower portions.

Without going into a detailed analysis of these calculations, we shall cite only those general data which may be used under practical operating conditions by a marine engineer for deriving the necessary conclusions relative to this problem.

1. Local bending of the plating. If the effective surfaces of crib-work and keel blocks in contact with the plating cover the span between frames, the specific pressure p_1 determined in conformity with Section 31 (column X, Table 19), must not exceed a known value capable of causing an inadmissible bending deflection of the plating. The value of the specific pressure, which is admissible insofar as the safety of the plating is concerned, must be considered as depending on the limiting depth of submersion of the submarine for which was calculated the strength of the plating.

In the nature of an approximate calculation, we may specify that the specific pressure must not surpass by more than 50% the pressure corresponding to the limit depth of submersion if there are no supplementary reinforcements in the form of an external keel and other braces or shorings reinforcing the plating. In conformity with the foregoing in general, the greatest specific pressures in drydocking submarines should be of an order of from 10 to 15 atm.

2. Bending of frames. The total pressure due to drydock reactions upon one span* appears as an external concentrated force which produces a bending of the circular frame and of the plating adjacent to it. This is

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*Editor's Note: The Russian here uses the word for "space". He may mean span or course.

counteracted by tangential stresses along the sections of the plating which limit the individual circular frame.

Thus, the bending of the frame is reduced to the bending of a ring, having a section consisting of the annular frame and the portion of the adjacent plating of a width equal to one span.

The largest bending moment in the lower section of the frame can be computed according to the formula:

$$M_{max} = \frac{3}{4\pi} Rr \qquad [6]$$

where R is the total pressure of the drydock reactions applying on one span; r is the radius of the circle of the frame.

The greatest stress in a frame can be found by the following formula:

$$\sigma_{max} = \frac{M_{max}}{W} \qquad [7]$$

where W is the moment of resistance of a section of the frame with the portion of the plating adjacent to it.

In checking the strength of the frame, according to Equation [7], the admissible stress must be taken in general as of the order of 60% of the yield point of the frame material.

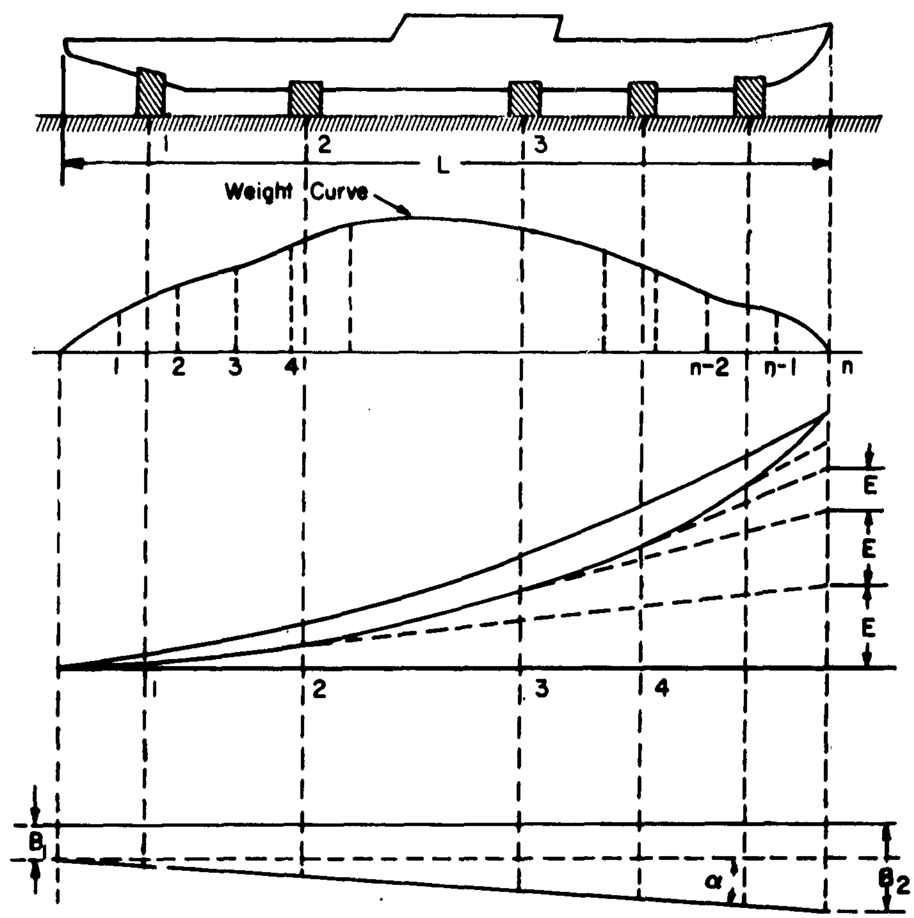

Figure 68

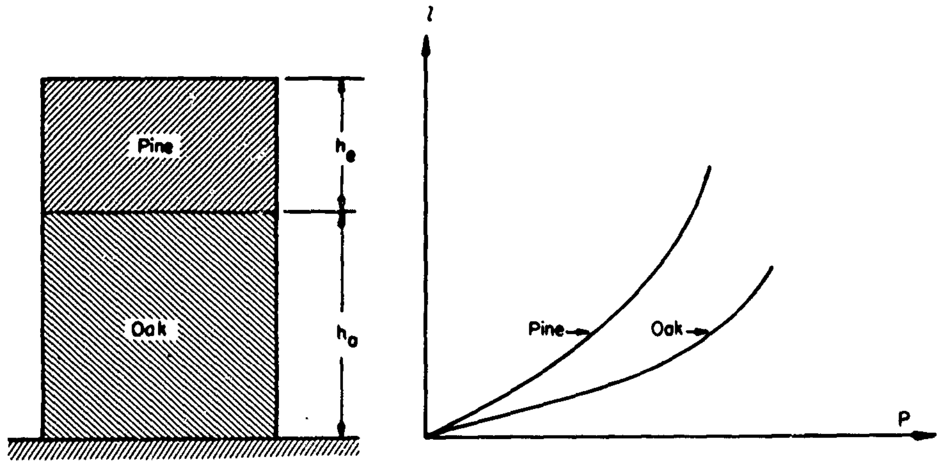

Figure 69

Table 18

Line No.	Computation of moments \ Weight curve ordinate number	0	1	2	......	n
1	Ordinates					
11	Sums of Line I in pairs					
111	Sums of Line II from Left					
1V	Sums of Line III in pairs					
V	Sums of Line 1V from Left					
	Ordinates of moment curves:* $1/4\ V\ (L : n)^2$ . .					

*Editor's Note: Not explained in text. The formula may mean: one-fourth of the values in Line V times the square of the quotient of L/n, where L is the length of the submarine and n is the value obtained from the columns of the table.

TABLE 19

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
							Drydock reaction				
Keel block	Distance (m)	Area (m^2)	Coefficient of rigidity (kg/cm^3)	Product $10 \cdot III \cdot IV$ (t/cm)	Product $II \cdot V$ ($t\text{-}m^2/cm$)	Product $II \cdot VI$ ($t\text{-}m^2/cm$)	Product $II \cdot \alpha$ (cm)	Sum $L_2 - VIII$ (cm)	Product $IV \cdot IX$ (kg/cm^2)	Product $III \cdot X$ (m)	Moments $II \cdot XI$ (t-m)
N_2	x_i	S_i	k_i	$10\ S_i\ k_i$	$10\ S_i\ k_i\ x_i$	$10\ S_i\ k_i\ x_i^2$	$\alpha\ x_i$	y_i	$p_i = k_i\ y_i$	$P_i = S_i\ p_i$	$P_i\ x_i$
1 2 . . . n											
Sum:		ΣS_i		A	B	C					

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assuming that the reader is familiar with the fundamentals of general structural mechanics, which are used in the book or which are referred to.

The book is subdivided into two parts; in the first, the general foundation and the practical methods of strength analysis of submarine hulls of various design are discussed; in the second, the theoretical investigation of those problems is presented whose solutions are used in the first part of the book and which were not contained in other sections of the general treatise, or were not sufficiently developed in those sections.

In writing the first part, which is essentially of applied character, the author made use of the material presented by him in the third volume of the "Handbook of Ship Construction." Several of the author's published theoretical papers devoted to questions of structural mechanics of submarines were included in the second part of the book.

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